

Mathematics

محاضرات في الرياضيات
كلية الهندسة
المرحلة الأولى

اعداد

الأستاذ الدكتور نزياد عبد الجليل الموسوي

٢٠١٧ / ٢٠١٨

Contents

chapter 0

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. احمد أ. د. ش. د. عبد الجليل

- 0.1 Inequalities
- 0.2 Functions (Domain and Range)
- 0.3 shifting graphs.
- 0.4 Trigonometric Functions.
- 0.5 Limits
- 0.6 Continuity

chapter 1

- 1.1 Differentiation.
- 1.2 Application of Differentiation
- 1.3 sketching graphs of polynomials and Rational Functions

chapter 2

- 2.2 Integration.
- 2.3 Application of definite integral
- 2.4 Area between two curves.
- 2.5 Volumes.
 - a) volumes by Disks.
 - b) volumes by washers.
 - c) volumes by cylindrical shells.
- 2.6 Length of curves.

chapter 3

- 3.1 Inverse Function
- 3.2 Logarithms.
- 3.3 The exponential Function.
- 3.4 The hyperbolic Function.
- 3.5 Inverse of trigonometric and hyperbolic functions

chapter 4

0.1 Inequalities

محاضرات في رياضيات
كلية الهندسة - أمّ البواقي
أ. د. منار عبد الجليل مجيب

Rules for Inequalities

If a, b and c are real numbers, then

1. $a < b \Rightarrow a + c < b + c$
2. $a < b \Rightarrow a - c < b - c$
3. $a < b$ and $c > 0 \Rightarrow ac < bc$
4. $a < b$ and $c < 0 \Rightarrow bc < ac$, special case $-b < -a$
5. $a > 0$ and $\frac{1}{a} > 0$
6. If a and b are both positive or both negative
then $a < b \Rightarrow \frac{1}{b} < \frac{1}{a}$

Ex1 Solve the following inequalities, and show their solution sets on the real line.

$$-\frac{x}{3} < 2x + 1$$

Sol $-x < 6x + 3 \Rightarrow -7x < 3 \Rightarrow x > -\frac{3}{7}$

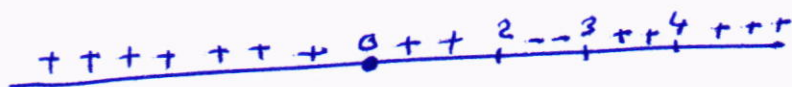


Ex2: Solve for x : $\frac{2x-5}{x-2} \leq 1$

Sol $\frac{2x-5}{x-2} - 1 \leq 0$

$$\frac{(2x-5) - (x-2)}{(x-2)} \leq 0$$

$$\frac{x-3}{x-2} \leq 0$$



\therefore Set of sol = $(2, 3]$

Functions, Domain and Range:

Def: A function f from a set D (Domain) to a set Y (Range) is any rule that assigns a unique

element $f(x) \in Y$ to each element $x \in D$

Ex1 Find the domain and the range of the following Function

$$2 - \sqrt{x} \geq 0$$

$$2 \geq \sqrt{x}$$

$$4 \geq x \Rightarrow 0 \leq x \leq 4$$

$$D_f : 0 \leq x \leq 4$$

$$\left. \begin{array}{l} \text{if } x=0 \Rightarrow y=\sqrt{2} \\ \text{if } x=4 \Rightarrow y=0 \end{array} \right\} R_f : 0 \leq y \leq \sqrt{2}$$

Note Any polynomial has the domain R , For ex
 $f(x) = \frac{1}{2}x^3 + 3x^2 - x + \pi$

Intervals:

1 The open interval is the set of all real numbers that be strictly between two fixed numbers a and b

$$(a, b) \equiv a < x < b$$



2 The closed interval is the set of all real numbers that contain both endpoints.

$$[a, b] \equiv a \leq x \leq b$$



3 Half open interval: is the set of all real numbers that contain one endpoint but not both

$$[a, b) \equiv a \leq x < b$$



$$(a, b] \equiv a < x \leq b$$



Absolute value:

The absolute value of a number x , denoted by $|x|$ is defined by the formula

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Absolute value properties..

1: $|-a| = |a|$

2: $|a \cdot b| = |a| \cdot |b|$

3: $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$

4: $|a+b| \leq |a| + |b|$

Ex:1 Solve the inequality involving absolute value

$$\left| 5 - \frac{2}{x} \right| < 1$$

Sol $\left| 5 - \frac{2}{x} \right| < 1$

$$\Rightarrow -1 < 5 - \frac{2}{x} < 1$$

$$\Rightarrow -6 < -\frac{2}{x} < -4$$

$$\Rightarrow 3 > \frac{1}{x} > 2$$

$$\Rightarrow \frac{1}{3} < x < \frac{1}{2}$$

Ex2 Solve:

$$|2x-3| \geq 1$$

Sol $2x-3 \geq 1$ or $2x-3 \leq -1$

$$x - \frac{3}{2} \geq \frac{1}{2} \text{ or } x - \frac{3}{2} \leq -\frac{1}{2}$$

$$x \geq 2, \text{ or } x \leq 1$$

The solution set $(-\infty, 1] \cup [2, \infty)$



فكرة الدالة
فكرة الدالة
فكرة الدالة

Composition of functions:

Suppose that the outputs of a function f can be used as inputs of a function g , we can then hook f and g together to form a new function whose inputs are the inputs of f and whose outputs are the numbers.

$$(g \circ f)_x = g(f(x)).$$

Ex: let $f(x) = \frac{x}{x-1}$ and $g(x) = 1 + \frac{1}{x}$ Find $(g \circ f)_x$ and $(f \circ g)_x$.

Sol $(g \circ f)_x = g(f(x)) = g\left(\frac{x}{x-1}\right) = 1 + \frac{1}{\frac{x}{x-1}} = \frac{2x-1}{x}$

$(f \circ g)_x = f(g(x)) = f\left(1 + \frac{1}{x}\right) = \frac{1 + \frac{1}{x}}{1 + \frac{1}{x} - 1} = x + 1$

Ex2 let $(g \circ f)_x = x$ and $f(x) = \frac{1}{x}$, Find $g(x)$

Sol $(g \circ f)_x = g\left(\frac{1}{x}\right) = x \Rightarrow g(x) = \frac{1}{x}.$

Increasing and decreasing Function:

Def: let f be a function defined on a interval I , and let x_1 , and x_2 be any two points in I .

1 if $f(x_2) > f(x_1)$, whenever $x_1 < x_2$, then f is said to be increasing on I

2 if $f(x_2) < f(x_1)$ whenever $x_2 < x_1$, then f is said to be decreasing on I .

Even Function and odd Function:

The graphs of even and odd functions have characteristic symmetry properties.

Def: A function $f(x)$ is an

Even function of x iff $f(-x) = f(x)$

Odd " " " iff $f(-x) = -f(x)$

عاشية بالترتيب
تكون/تكون
في زوجي بالترتيب

Note The graph of an odd function is symmetric about the origin, since $f(-x) = -f(x)$

Ex: Even function: $f(x) = x^2$ is even function symmetry about y -axis.

Ex: odd function, $f(x) = x+1$, Not odd $f(-x) = -x+1$ but $-f(x) = -x-1$, The two are not equal.

Types of Functions: -

1. Linear Function:

$$f(x) = mx + b.$$

2. power function

$$f(x) = x^a, \quad a \text{ is a constant}$$

3. polynomials:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0.$$

4. Rational Function

$$f(x) = \frac{p(x)}{q(x)}, \quad q(x) \neq 0.$$

Ex: $f(x) = \frac{2x^2 - 3}{7x + 4}$

5. Algebraic Function.

$$y = \frac{3}{4} (x^2 - 1)^{2/3}$$

6. Trigonometric function:

$$f(x) = \sin x, \quad f(x) = \cos x$$

7. Exponential function.

$$f(x) = a^x \text{ have domain } (-\infty, \infty), \text{ Range } (0, \infty)$$

8 Logarithmic Function:

$$f(x) = \log_a x, \quad a \neq 1, a > 0$$

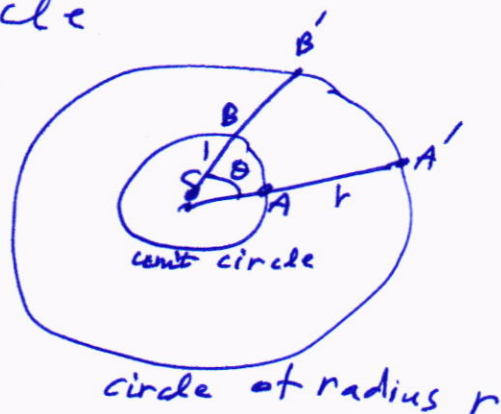
مضاعفات في الرياضيات
كلية الهندسة / المهندسين
مهناشاد أود نجاد عبد الله

Trigonometric Function:

The radian measure of the Angle at the center of the unit circle as shown in the Fig equals the length of the arc that ACB cuts from the unit circle

Thus $s = r\theta$ is the length of arc on a circle of radius r when θ is measured in radians

$$\frac{s}{r} = \frac{\theta}{1} \Rightarrow s = r \cdot \theta$$



Note if The circle is a unit circle having radius = 1
The one complete revolution of the unit circle is 360° or 2π radians, we have

$$\pi \text{ radians} = 180^\circ$$

$$2\pi \text{ radians} = 360^\circ$$

$$1 \text{ degree} = \frac{\pi}{180}$$

degrees to radians multiply by $\frac{\pi}{180}$

$$1 \text{ radians} = \frac{180}{\pi}$$

Radians to degree, multiply by $\frac{180}{\pi}$

For ex 45° is radian measure is

$$45 \cdot \frac{\pi}{180} = \frac{\pi}{4},$$

$$\text{and } \pi/6 \text{ radians is } \frac{\pi}{6} \cdot \frac{180}{\pi} = 30^\circ$$

The Six Basic Trigonometric Functions

$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$

$$\sec \theta = \frac{r}{x}$$

$$\csc \theta = \frac{r}{y}$$

$$\tan \theta = \frac{1}{\cot \theta}, \quad \cos \theta = \frac{1}{\sec \theta}, \quad \sin \theta = \frac{1}{\csc \theta}.$$

$$x = r \cos \theta, \quad y = r \sin \theta.$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

Addition and Subtract Formulas:

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta.$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1$$

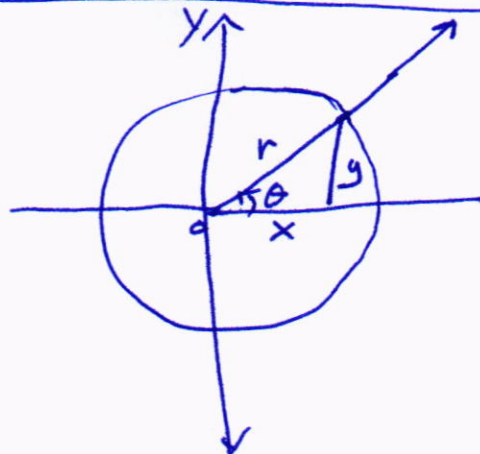
$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}.$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

ماضیہ الراجحہ
کتبہ المکتبہ / المکتبہ الاولی
نہ انوار
آؤد نیواری اللہ



$$\begin{aligned}\sin(\theta \pm 2\pi) &= \sin \theta \\ \cos(\theta \pm 2\pi) &= \cos \theta \\ \sec(\theta \pm 2\pi) &= \sec \theta\end{aligned}$$

$\left. \begin{aligned}\tan(\theta \pm 2\pi) &= \tan \theta \\ \cot(\theta \pm 2\pi) &= \cot \theta \\ \csc(\theta \pm 2\pi) &= \csc \theta\end{aligned} \right\}$
 تماميات يا ارباعيات
 كليه اطياف / الزوايا الاول
 ماركاد ا. د. نرعد عبد الباق

$$\left. \begin{aligned}\cos(-x) &= \cos x \\ \sec(-x) &= \sec x\end{aligned} \right\} \text{ even}$$

$$\left. \begin{aligned}\sin(-x) &= -\sin x \\ \tan(-x) &= -\tan x \\ \csc(-x) &= -\csc x \\ \cot(-x) &= -\cot x\end{aligned} \right\} \text{ odd}$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos(x)$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin(x)$$



Limits

محاضرة في الرياضيات
كتبه الأستاذ
م. أ. د. نور عبد الله

if $f(x)$ is approaches to L , when x approaches to x_0 from left and right then we say that the limit exist at $x = x_0$

or $\lim_{x \rightarrow x_0} f(x) = L$

if $\lim_{x \rightarrow x_0^+} f(x) = \lim_{x \rightarrow x_0^-} f(x) = L$.

Theorem: if $\lim_{x \rightarrow a} f(x) = L_1$, $\lim_{x \rightarrow a} g(x) = L_2$, and k

is constant then:

1) $\lim_{x \rightarrow a} (f(x) \pm g(x)) = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) = L_1 \pm L_2$

2) $\lim_{x \rightarrow a} k f(x) = k \lim_{x \rightarrow a} f(x) = k \cdot L_1$

3) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{L_1}{L_2}$, $L_2 \neq 0$

4) $\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)} = \sqrt[n]{L_1}$, $L_1 \geq 0$

Ex Find $\lim_{x \rightarrow 5} (x^2 - 4x + 3) = 5^2 - 4 \times 5 + 3 = 8$

Ex Find $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{(x-2)}$

$= \lim_{x \rightarrow 2} (x+2) = 4$

Ex: $\lim_{x \rightarrow 4} \frac{2-x}{(x-4)(x+2)}$

Sol $\lim_{x \rightarrow 4^-} \frac{2-x}{(x-4)(x+2)} = -\infty$

$\lim_{x \rightarrow 4^+} \frac{2-x}{(x-4)(x+2)} = +\infty$

does not exist

H.W Find

1 $\lim_{x \rightarrow -\infty} \frac{4x^2 - x}{2x^3 - 5}$

Ans: 0

2 $\lim_{x \rightarrow +\infty} \frac{3x+5}{6x+8}$

Ans: $\frac{1}{2}$

3 $\lim_{x \rightarrow -\infty} \sqrt[3]{\frac{3x+5}{6x-8}}$

Ans: $\sqrt[3]{\frac{1}{2}}$

4 $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2-2}}{3x-6}$

Ans: $-\frac{1}{3}$

5: $\lim_{x \rightarrow 3} f(x) = \begin{cases} x^2-5, & x \leq 3 \\ \sqrt{x+13}, & x > 3 \end{cases}$ Ans: 4

Ex: Find $\lim_{x \rightarrow 0} \frac{x}{|x|}$

$= \lim_{x \rightarrow 0} \frac{x}{|x|} = \begin{cases} \frac{x}{x}, & x \geq 0 \\ \frac{x}{-x}, & x < 0 \end{cases} = \begin{cases} 1, & x \geq 0 \\ -1, & x < 0 \end{cases}$

The L. does not exist

Ex: Find $\lim_{x \rightarrow 1} \frac{x^3 + x^2 - 5x + 3}{x^3 - 3x + 2}$ Ans: $\frac{4}{3}$

Theorem

1 $\lim_{x \rightarrow 0} \frac{\sin ax}{ax} = 1, a \neq 0$

2 $\lim_{x \rightarrow 0} \sin x = 0$

3 $\lim_{x \rightarrow 0} \cos x = 1$

4 $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$

5 $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$

6 $\lim_{x \rightarrow \infty} \frac{\cos x}{x} = 0$

$$\text{Ex } \lim_{x \rightarrow \infty} \sin\left(\frac{\pi x}{2-3x}\right)$$

$$= \sin \lim_{x \rightarrow \infty} \frac{\pi x}{2-3x}$$

$$= \sin \lim_{x \rightarrow \infty} \left(\frac{\pi}{\frac{2}{x} - 3} \right) = \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

محاضرات في الرياضيات
كتبه المهندس / المرحوم الأستاذ
أ. د. محمد عبد الجليل

$$\text{Ex: } \lim_{x \rightarrow \pi} \frac{(\pi-x)}{\sin x} = \lim_{x \rightarrow \pi} \frac{(\pi-x)}{\sin(\pi-x)}$$

Let $\pi-x=t \Rightarrow x=\pi-t$, if $x \rightarrow \pi$ then $t \rightarrow 0$

$$= \lim_{t \rightarrow 0} \frac{t}{\sin t} = \lim_{t \rightarrow 0} \frac{t/t}{\frac{\sin t}{t}} = 1$$

$$\text{Ex: } \lim_{x \rightarrow \infty} \frac{2x + x \sin x}{5x^2 - 2x + 1}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{2x}{5x^2} + \frac{x \sin x}{x^2}}{\frac{5x^2}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2}} = \frac{\lim_{x \rightarrow \infty} \frac{2}{5x} + \lim_{x \rightarrow \infty} \frac{\sin x}{x}}{\lim_{x \rightarrow \infty} 5 - \lim_{x \rightarrow \infty} \frac{2}{x} + \lim_{x \rightarrow \infty} \frac{1}{x^2}} = 0$$

(Continuity)

Definition:

A function f is said to be continuous at a point c , if the following conditions are satisfied:

1. $f(c)$ is defined.

2. $\lim_{x \rightarrow c} f(x)$ exists

3. $\lim_{x \rightarrow c} f(x) = f(c)$

If one or more conditions in this definition fails to hold, then f is called discontinuous at c , and c is called the point of discontinuity of f .

$$\text{Ex Let } f(x) = \frac{x^2 - 4}{x - 2}$$

→ The function f at $x=2$ is undefined, so that f is discontinuous.

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
أ. د. نزار عبد الجليل محمد

Ex Does the function $f(x) = \begin{cases} 2x+3, & x \leq 4 \\ 7 + \frac{16}{x}, & x > 4 \end{cases}$

continuous at $x=4$

1. $f(4) = 2 \times 4 + 3 = 11$ defined at $x=4$

2. $\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^+} (7 + \frac{16}{x}) = 11$ exist.

3. $\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^-} (2x+3) = 11$

4. $\lim_{x \rightarrow 4} f(x) = f(4) = 11$

Theorem. If the functions f and g are continuous at c , then:

1. $f+g$ is continuous at c .

2. $f-g$ " " " c

3. $f \cdot g$ " " " c

4. f/g " " " c , if $g(c) \neq 0$ and g is continuous at c , if $g(c) = 0$.

Ex where is $h(x) = \frac{x^2-9}{x^2-5x+6}$ continuous

Sol $x^2-5x+6 = (x-3)(x-2)$

$x=2, x=3$

Hence $h(x)$ is continuous everywhere except at these points $(2, 3)$

Theorem. If f and g are continuous at c , then $f \circ g$ is continuous at c .

H.W

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. اسامه ا. د. محمد عبد الجليل

1 Find $\lim_{x \rightarrow c} \frac{x^4 + x^2 - 1}{x^2 + 5}$, Ans $\frac{c^4 + c^2 - 1}{c^2 + 5}$

2 $\lim_{x \rightarrow -2} \sqrt{4x^2 - 3}$, Ans: $\sqrt{13}$

3 $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x}$, Ans: 3

4 Suppose $\lim_{x \rightarrow c} \frac{f(x)}{f(x) - g(x)}$, if $\lim_{x \rightarrow c} f(x) = 5$, $\lim_{x \rightarrow c} g(x) = -2$

Find $\lim_{x \rightarrow c} \frac{f(x)}{f(x) - g(x)}$, Ans $\frac{5}{7}$

chapter 1 Differentiation

Definition: Let $y = f(x)$ be a function, then the derivative of f with respect to x is defined by $\frac{dy}{dx} = y' = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.

Theorem: If f and g are differentiable function at x and k is constant then.

1. $\frac{d}{dx} (k) = 0$

2. $\frac{d}{dx} (f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

3. $\frac{d}{dx} (f(x) \cdot g(x)) = f(x) \cdot \frac{d}{dx} g(x) + g(x) \cdot \frac{d}{dx} f(x)$

4. $\frac{d}{dx} (k f(x)) = k \frac{d}{dx} f(x)$

5. $\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) \cdot \frac{d}{dx} f(x) - f(x) \cdot \frac{d}{dx} g(x)}{(g(x))^2}$, provided $g(x) \neq 0$

6. $\frac{d}{dx} x^n = n x^{n-1}$

Ex: Let $y = \left(\frac{x-1}{x+1}\right)(2x^7 - x^2)$, Find y'

محاضرات في الرياضيات
كلية الهندسة المرحلة الأولى

Sol $y' = \left(\frac{x-1}{x+1}\right)(14x^6 - 2x) + (2x^7 - x^2) \frac{(x+1) \cdot 1 - x(x+1)}{(x+1)^2}$

$$= \left(\frac{x-1}{x+1}\right)(14x^6 - 2x) + \frac{2(2x^7 - x^2)}{(x+1)^2}$$

Ex2: Find y'' , where $y = 6x^5 - 4x^2$

Sol $y' = 30x^4 - 8x$

$$y'' = 120x^3 - 8$$

$$y''_{at 1} = 112$$

Derivatives of Trigonometric Function:

1 $y = \sin x \Rightarrow y' = \cos x$

2 $y = \cos x \Rightarrow y' = -\sin x$

3 $y = \tan x \Rightarrow y' = \sec^2 x$

4 $y = \cot x \Rightarrow y' = -\csc^2 x$

5 $y = \sec x \Rightarrow y' = \sec x \tan x$

6 $y = \csc x \Rightarrow y' = -\csc x \cot x$

Chain Rule:

1 if $y = f(u)$, $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Ex: Let $y = u^{10}$, and $u = x^3 + 7x + 1$, Find y' .

Sol: $\frac{dy}{du} = 10u^9$, $\frac{du}{dx} = 3x^2 + 7 \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$$\frac{dy}{dx} = (10x^9) \cdot (3x^2 + 7)$$

$$\frac{dy}{dx} = [10(x^3 + 7x + 1)^9] [2x^2 + 7]$$

طريقة ثانية

$$y = (x^3 + 7x + 1)^{10} \Rightarrow \frac{dy}{dx} = [10(x^3 + 7x + 1)^9] \cdot [2x^2 + 7]$$

Implicit Differentiation:

Ex: Find y' , if $y^2 = x$

Sol $y^2 = x$

$$2y \cdot y' = 1$$

$$y' = \frac{1}{2y}$$

Ex: Find y' , if $2x^3 - 3y^2 = 8$

Sol $6x^2 - 6yy' = 0$

$$6yy' = 6x^2$$

$$yy' = x^2 \Rightarrow y' = \frac{x^2}{y}, \text{ when } y \neq 0.$$

$$3y^2 = 2x^3 - 8 \Rightarrow y^2 = \frac{2x^3 - 8}{3}, \quad y = \pm \sqrt{\frac{2x^3 - 8}{3}}$$

$$\therefore y' = \frac{x^2}{\pm \sqrt{\frac{2x^3 - 8}{3}}}$$

H.W

1 Find y' if $y^2 = x^2 + \sin xy$, Ans: $y' = \frac{2x + y \cos xy}{2y - x \cos xy}$

2 Find y' , if $x^2 + y^3 - 9xy = 0$, Ans: $y' = \frac{2y - x^2}{y^2 - 9x}$

3 Find y' , if $y = x^2y + xy^2 = 6$, Ans: $\frac{-2xy - y^2}{x^2 + 2xy}$

4: Find y' , if $x = \tan y$, Ans: $\cos^2 y$.

Application of Differentiation

حافظات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. اسد أ. د. زياد عبد الله

Theorem:

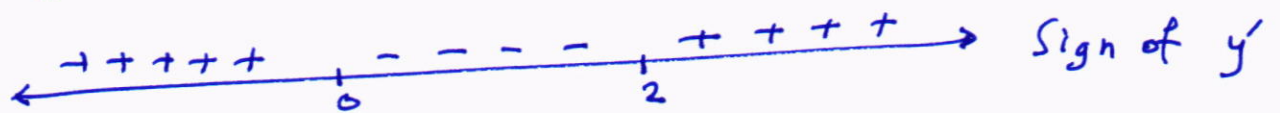
1. if $f'(x) > 0$ on (a, b) , then f is increase on (a, b) .
2. if $f'(x) < 0$ on (a, b) then f is decrease on (a, b)

Ex: On which intervals is the function :

$f(x) = x^3 - 3x^2 + 1$, increase or decrease.

Sol $f'(x) = 3x^2 - 6x$
 $= 3x(x - 2) = 0$.

$x = 0$ or $x = 2$



So $f(x)$ is increase on $(-\infty, 0) \cup (2, \infty)$
and decrease on $(0, 2)$.

Def: A function $f(x)$ is concave up on (a, b)

if $y'' > 0$

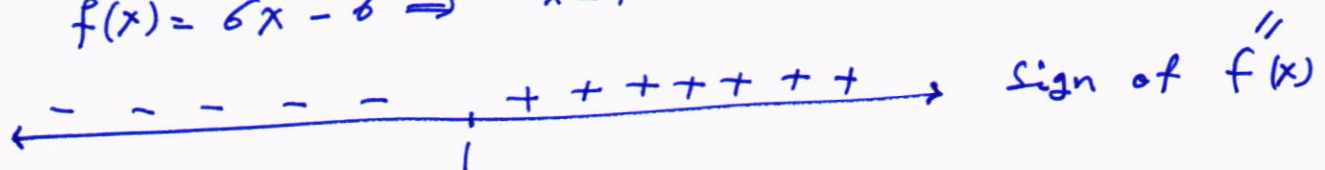
and concave down on (a, b)

if $y'' < 0$.

Ex: Find the interval where the function
 $f(x) = x^3 - 3x^2 + 1$ concave up, concave down

Sol $f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6 \Rightarrow x = 1$



f is concave up on $(1, \infty)$, and concave down on $(-\infty, 1)$

Def: let f be a continuous function on $[a, b]$, and f

changes direction of concavity at x_0
 then $(x_0, f(x_0))$ called inflection
 point of f .

محاذات في الرافعة
 كتيه الطنف / المرفعة الأولى
 من اعداد ا. د. نورا عبد الطيل

or $(x_0, f(x_0))$ is an inflection point if $y''(x) = 0$.

Ex: Find the Location of all inflection points of:

$$f(x) = x^4 - 8x^2 + 16$$

$$f'(x) = 4x^3 - 16x$$

$$f''(x) = 12x^2 - 16 \Rightarrow 4(3x^2 - 4) = 0$$

$$x = \pm \frac{2}{\sqrt{3}}$$

$(\frac{2}{\sqrt{3}}, 7.1)$ and $(\frac{-2}{\sqrt{3}}, 7.1)$ are inflection points.

Ex: Find the intervals in which the function:

$$f(x) = x^{\frac{1}{3}}(x+4)$$

Is increasing, decreasing, concave up, concave down
 and inflection point.

Sol $f(x) = x^{\frac{1}{3}}(x+4)$

$$f'(x) = y' = x^{\frac{1}{3}} \cdot 1 + (x+4) \left(\frac{1}{3}\right) x^{-2/3}$$

$$f'(x) = x^{\frac{1}{3}} + \frac{x+4}{3x^{2/3}}$$

$$= \frac{4x+4}{3x^{2/3}}$$

← - - - - - | + + + + + → Sign of y'
 -1 0

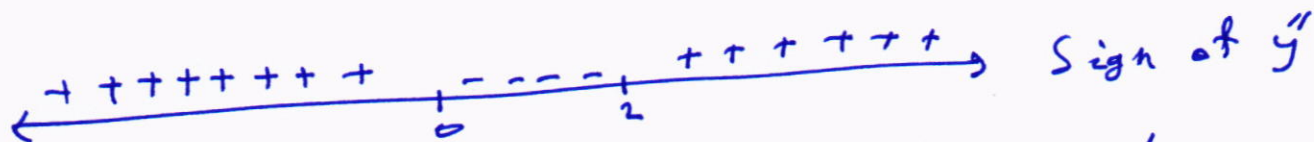
f increase on $(-1, 0)$, $(0, \infty)$

f decrease on $(-\infty, -1)$

$$f'(x) = \frac{4}{3}(x+1) \left(\frac{-2}{3}x^{-5/2}\right) + x^{-2/3}$$

$$\frac{4(x-2)}{9x^{5/3}} = 0 \Rightarrow x = 2$$

محاضرات في الرياضيات
تلك المهندسة / المرحلة الأوردة
منه أعداد أ. د. نجاد عبد المولى



f concave up on $(-\infty, 0)$, $(2, \infty)$ concave down on $(0, 2)$, $(2, 2.6)$ is the inflection point.

H.W.

1. On which intervals is the function: $f(x) = x^3 - 3x^2 + 1$, increasing, decreasing
Ans: inc if $x < 0$
dec if $0 < x < 2$
inc if $x > 2$

2. Where is the function $f(x) = x^3 - 3x^2 + 1$, concave up, concave down?
Ans: f concave up if $x > 1$
 f " down if $x < 1$

3. Determine open intervals on which f is
a) increasing, b) decreasing, c) concave up, d) concave down
e) Find the location of all inflection points.
 $f(x) = x^2 - 5x + 6$

Ans:

a) $(5/2, \infty)$, b) $(-\infty, 5/2)$, c) $(-\infty, \infty)$, d) (none)
e) (none)

4. Find the inflections points, if any, of the function $f(x) = (x-a)^4$
Ans: none

5. prove that:

a) $f(x) = x^2$ is increasing on $(0, \infty)$
b) $f(x) = x^2 - 2x$ is decreasing on $(-\infty, 1]$

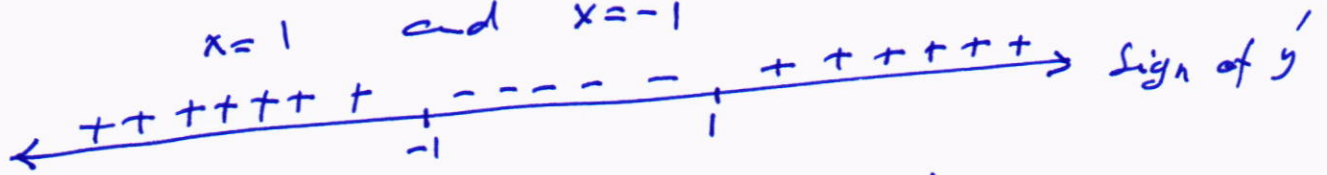
Sketching graphs

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. اسد أ. د. نزيه عبد الحليم

Ex: sketch the graph of the curve $f(x) = x^3 - 3x + 2$

Sol $f'(x) = 3x^2 - 3 = 3(x-1)(x+1) = 0$

$x = 1$ and $x = -1$



f increase on $(-\infty, -1), (1, \infty)$

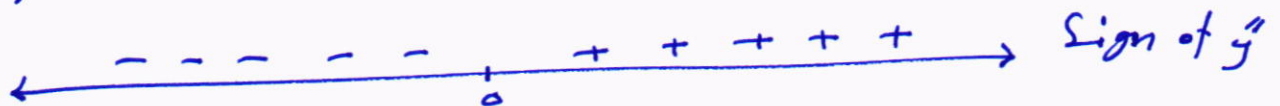
f decrease on $(-1, 1)$

$f(-1) = 4 \Rightarrow (-1, 4)$ is relative maximum

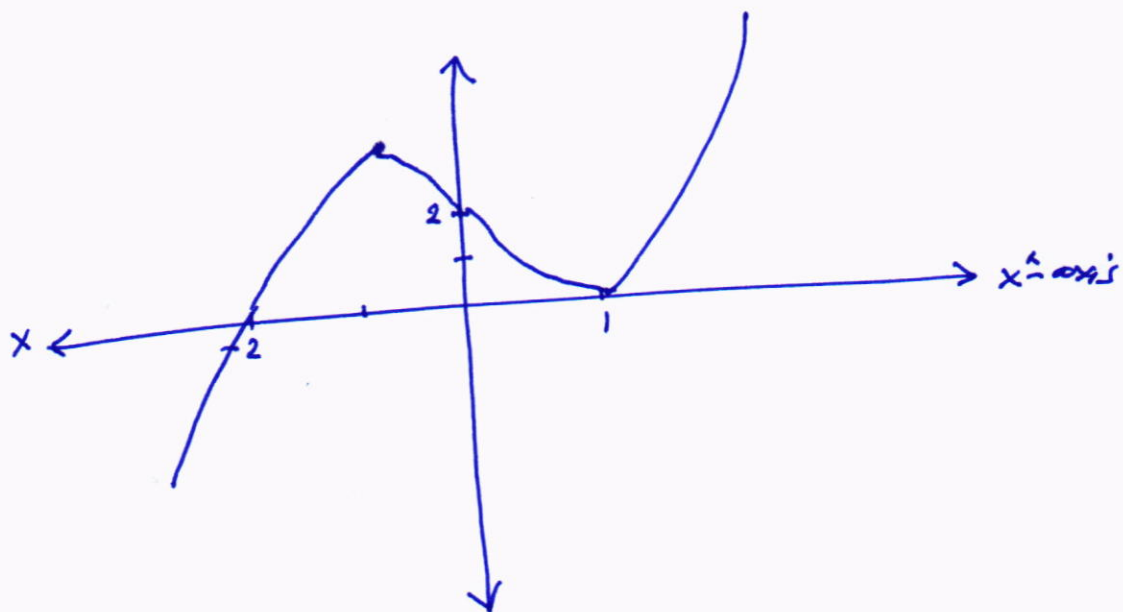
$f(1) = 0 \Rightarrow (1, 0)$ is relative minimum

$f' \begin{cases} - & \rightarrow + \\ + & \rightarrow \max \end{cases}$

$f''(x) = 6x = 0 \Rightarrow x = 0$
 $(0, 2)$ is inflection point.



f is concave up on $(0, \infty)$, and concave down on $(-\infty, 0)$



Exercise

a) Find values of the constants a, b and c that will make

$$f(x) = \cos x, \text{ and } g(x) = a + bx + cx^2$$

satisfy the conditions.

$$f(0) = g(0), \quad f'(0) = g'(0), \quad \text{and} \quad f''(0) = g''(0)$$

$$\text{Ans: } a = 1, \quad b = 0, \quad c = -\frac{1}{2}$$

b) Find values for b and c that will make

$$f(x) = \sin(x+a), \text{ and } g(x) = b \sin x + c \cos x$$

satisfy the conditions

$$f(0) = g(0) \quad \text{and} \quad f'(0) = g'(0).$$

$$\text{Ans: } b = \cos a, \quad c = \sin a$$

c) For the determined values of a, b and c , what happens for the third and fourth derivatives of f and g in each parts a and b.



Chapter 2

Def. $\frac{d}{dx}[f(x)] = f'(x)$ then $\int f'(x) dx = f(x) + c$

Def: if $y = f(x)$ is nonnegative and integrable over closed interval $[a, b]$, then the area under the curve $y = f(x)$ over $[a, b]$ is the integral of f from a to b

$$A = \int_a^b f(x) dx.$$

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
مهاجدة ١٤٠٥ هـ / محمد عبد الجليل

$$1. \int c \cdot f(x) dx = c \int f(x) dx$$

$$2. \int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

$$3. \int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$$

4. $\int \cos u \, du = \sin u + C$

$$5. \int \sin u \, du = -\cos u + C$$

6. $\int \sec^2 u \, du = \tan u + C$

$$7. \int \csc^2 u \, du = -\cot u + C$$

$$\int \sec u \tan u \, du = \sec u + C$$

8. $\int \csc u \cot u \, du = -\csc u + C$

Examples:

Ex1 Evaluate $\int 4 \cos x \, dx$

Ex1

Sol. $\int 4 \cos x \, dx = 4 \int \cos x \, dx$
 $= 4 (\sin x + c)$
 $= 4 \sin x + 4c$

$$E_{x_2} \int (x + x^2) dx$$

Sol: $\int (x+x^2) dx = \int x dx + \int x^2 dx$
 $= \frac{x^2}{2} + \frac{x^3}{3} + c_1 + c_2$
 $= \frac{x^2}{2} + \frac{x^3}{3} + c$

Ex 3 $\int \frac{x^2 - 2x^4}{x^4} dx$

Sol: $\int (\frac{1}{x^2} - 2) dx \Rightarrow \int (x^{-2} - 2) dx = \frac{x^{-1}}{-1} - 2x + C$

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
مهنا محمد أ. د. محمد عبد الحليم

Ex Evaluate

$$\int (x^2 + 1)^{50} 2x \cdot dx$$

Sol let $u = x^2 + 1$

$$\Rightarrow \frac{du}{dx} = 2x$$

$$\Rightarrow \frac{du}{dx} = 2x$$

$$\int (x^2+1)^{50} \cdot 2x \cdot dx = \int \left[u^{50} \frac{du}{dx} \right] \cdot dx = \int u^{50} du$$

$$= \frac{u^{51}}{51} + C = \frac{(x^2+1)^{51}}{51} + C.$$

Ex: Evaluate

$$\int \frac{\cos \sqrt{x}}{2\sqrt{x}} dx$$

Sol: let $u = \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow du = \frac{1}{2\sqrt{x}} dx$

Soln: Let $u = \sqrt{x}$ then $du = \frac{1}{2\sqrt{x}} dx$

$$\int \frac{\cos \sqrt{x}}{2\sqrt{x}} dx = \int \cos u \, du = \sin u + C = \sin \sqrt{x} + C$$

Ex Evaluate:

$$\int \sin(x+9) dx$$

Sol.: let $u = x+9$
 $du = dx$

$$\int \sin(x+9) dx \Rightarrow \int \sin u du = -\cos u + C$$
$$\Rightarrow -\cos(x+9) + C$$

Ex: Evaluate:

$$\int \cos 5x \, dx \Rightarrow \int (\cos u) \cdot \frac{1}{5} \, du = \frac{1}{5} \int \cos u \, du$$

$$u = 5x \Rightarrow du = 5dx$$

$$\int \cos 5x \, dx = \frac{1}{5} \sin u + C \implies \frac{1}{5} \sin 5x + C$$

The definite integral:

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
مهامد أ. د. نجاد عبد الباق

Ex: Evaluate

$$\int_0^3 (x^3 - 4x + 1) dx$$
$$= \left[\frac{x^4}{4} - 4 \frac{x^2}{2} + x \right]_0^3 = \left(\frac{81}{4} - 18 + 3 \right) - 0 = \frac{21}{4}$$

Properties:

1) $\int_a^a f(x) dx = 0$

2) $\int_b^a f(x) dx = - \int_a^b f(x) dx$

3) $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$
if $a < c < b$

Ex. $\int_5^5 x^3 dx$

Sol $\int_5^5 x^3 dx = \left[\frac{x^4}{4} \right]_5^5 = \frac{625}{4} - \frac{625}{4} = 0.$

Ex: If $\int_0^x \frac{\sin t}{t} dx$, Find $F'(x)$

$$F'(x) = \frac{\sin x}{x}$$



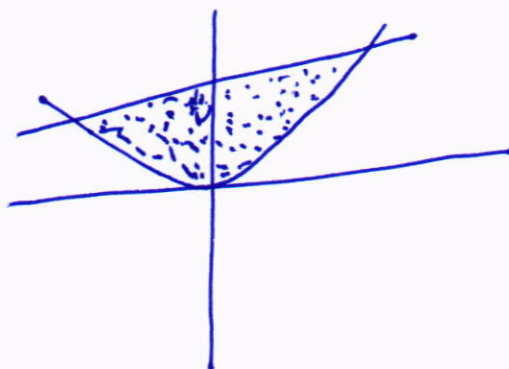
محاضرات في الرياضيات
لكية الهندسة المرحلة الأولى
من إعداد د. محمد عبد الجليل

a if $f(x) \geq g(x)$, for $a \leq x \leq b$ then:

b if $f(y) \geq g(y)$, for $c \leq y \leq d$ then:

$$A = \int_c^d [C f(y) - g(y)] dy.$$

the curves $y_1 = x^2$ and $y_2 = x + 6$.



$$x^2 - x - 6 = 0$$

$$(x+2)(x-3)=0$$

$$\therefore x = -2, \quad x = 3$$

check $x=0$.

here $x=0$:
 $y_1 = 0$ (below), $y_2 = 6$ (above)

$$A = \int_{-2}^3 (y_2 - y_1) dx = \int_{-2}^3 [(x+6) - x^2] dx$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3} \right]_{-2}^3$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3} \right]_{-2}^3 = \left(\frac{3^2}{2} + 6 \times 3 - \frac{3^3}{3} \right) - \left(\frac{(-2)^2}{2} + 6 \times (-2) - \frac{(-2)^3}{3} \right)$$

$$= \frac{27}{2} - \left(-\frac{22}{3}\right) = \frac{125}{6} \text{ units}^2$$

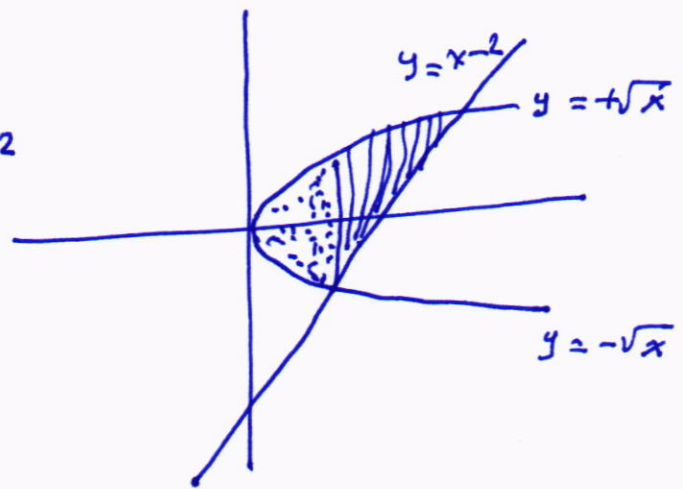
Ex Find the area of the region enclosed between the curves:
 $x = y^2$ and $y = x - 2$

محاضرات في الرياضيات
 كلية الهندسة / المرحلة الأولى
 ص. اعداد أ. د. نجاد عبد المطلب

Method (1):

$$y_1 = \sqrt{x}, \quad y_2 = -\sqrt{x}, \quad y_3 = x - 2$$

$$\begin{cases} \sqrt{x} = -\sqrt{x} \\ \sqrt{x} + \sqrt{x} = 0 \\ 2\sqrt{x} = 0 \\ \sqrt{x} = 0 \\ x = 0 \end{cases} \begin{cases} \pm\sqrt{x} = x - 2 \\ x = (x - 2)^2 \\ x^2 - 5x + 4 = 0 \\ (x - 1)(x - 4) = 0 \\ \therefore x = 1, x = 4 \end{cases}$$



$$A = A_1 + A_2$$

$$A = \int_0^1 [\sqrt{x} - (-\sqrt{x})] dx + \int_1^4 [\sqrt{x} - (x - 2)] dx$$

$$= \int_0^1 2\sqrt{x} dx + \int_1^4 [\sqrt{x} - x + 2] dx$$

$$= 2 \int_0^1 x^{\frac{1}{2}} dx + \int_1^4 \left[x^{\frac{1}{2}} - x + 2 \right] dx$$

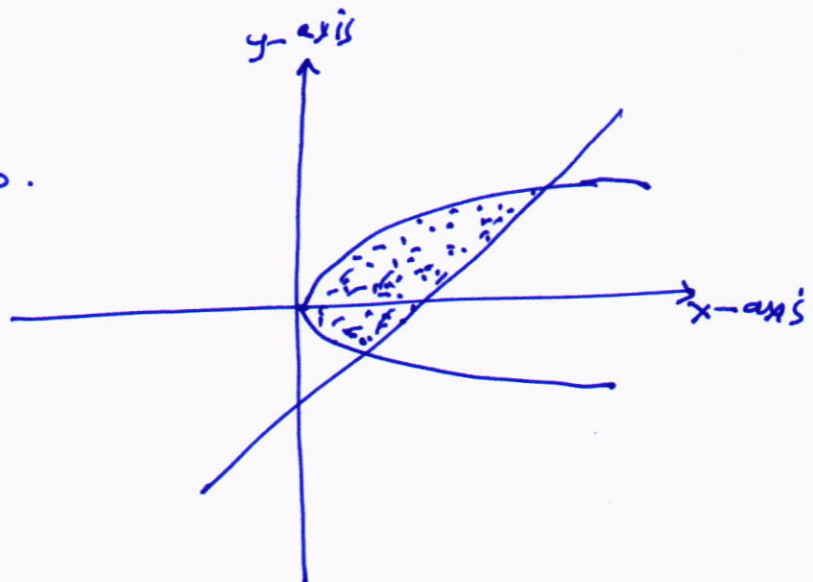
$$= 2 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 + \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{2} + 2x \right]_1^4 = \frac{4}{3} + \frac{19}{6} = \frac{9}{2}$$

Method (2)

$$y^2 = y + 2, \quad y^2 - y - 2 = 0.$$

$$(y + 1)(y - 2) = 0$$

$$y = -1, \quad y = 2$$



$$A = \int_{-1}^2 [(y+2) - y^2] dy$$

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
مه امداد أ. د. نوار عبد الجليل

$$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_{-1}^2$$

$$= \left[\frac{2^2}{2} - 2 \times 2 - \frac{2^3}{3} \right] - \left[\frac{(-1)^2}{2} - 2(-1) - \frac{(-1)^3}{3} \right] = \frac{9}{2} \text{ units}^2$$

Ex: Find the area of the region enclosed by the
Parabola $y = 2 - x^2$ and the line $y = -x$.

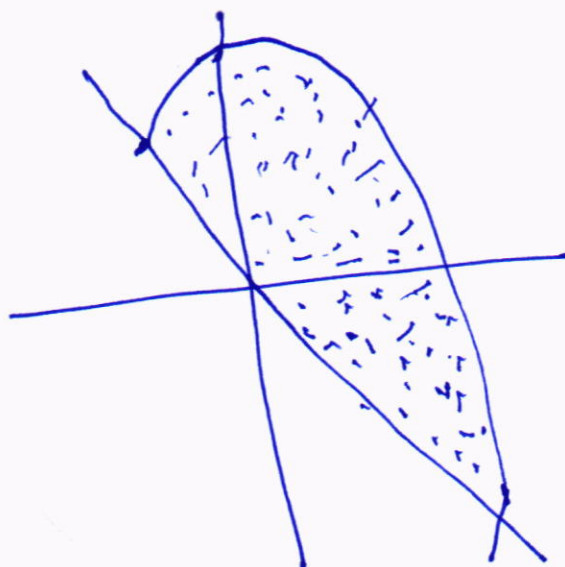
Sol

$$2 - x^2 = -x$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = -1, x = 2$$



$$A = \int_a^b [f(x) - g(x)] dx$$

$$= \int_{-1}^2 [(2 - x^2) - (-x)] dx$$

$$= \int_{-1}^2 (2 + x - x^2) dx = \left[2x + \frac{x^2}{2} - \frac{x^3}{3} \right]_{-1}^2$$

$$= \left(4 + \frac{4}{2} - \frac{8}{3} \right) - \left(-2 + \frac{1}{2} + \frac{1}{3} \right) = \frac{9}{2} \text{ units}^2$$

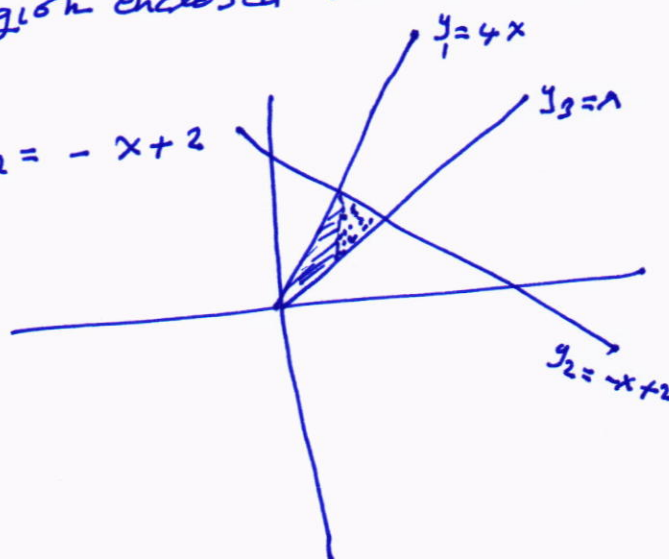
Ex: Find the area of the region enclosed between the
curves.

$$y_3 = x, \quad y_1 = 4x, \quad y_2 = -x + 2$$

$$y_1 = y_2$$

$$4x = -x + 2$$

$$5x = 2 \Rightarrow x = \frac{2}{5}$$



$$y_1 = y_2$$

$$4x = 0 \Rightarrow x = 0$$

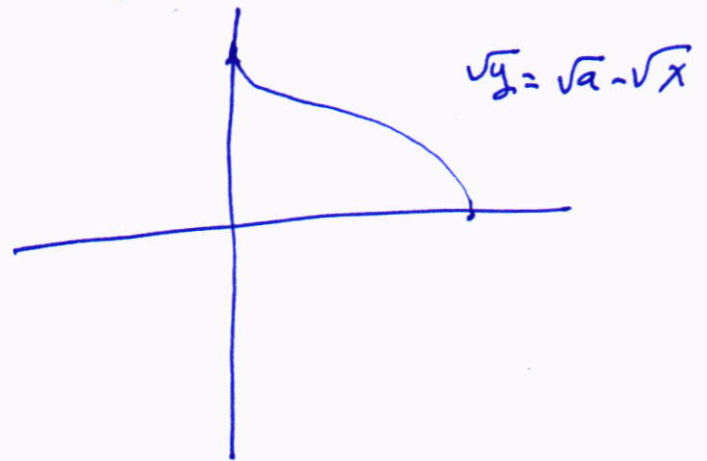
$$A = A_1 + A_2$$

$$A_2 = \int_0^{2/5} (4x - x) dx + \int_{2/5}^1 [(-x + 2) - x] dx$$

$$= \left[4 \frac{x^2}{2} - \frac{x^2}{2} \right]_0^{2/5} + \left[-\frac{x^2}{2} + 2x - \frac{x^2}{2} \right]_{2/5}^1$$

$$= \left[\frac{3x^2}{2} \right]_0^{2/5} + \left[-x^2 + 2x \right]_{2/5}^1 = \frac{3}{5}$$

Ex: Find the area bounded by $\sqrt{x} + \sqrt{y} = \sqrt{a}$, and the coordinate axes



Sol $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Put $x = 0$

$$\sqrt{y} = \sqrt{a}$$

$$y = a \Rightarrow (0, a)$$

Put $y = 0$

$$\sqrt{x} = \sqrt{a}$$

$$x = a \Rightarrow (a, 0)$$

$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$

$$\sqrt{y} = \sqrt{a} - \sqrt{x}$$

$$y = (\sqrt{a} - \sqrt{x})^2 \Rightarrow y = a - 2\sqrt{a}\sqrt{x} + x$$

$$\therefore A = \int_0^a (a - 2\sqrt{a}\sqrt{x} + x) dx = \left[ax - 2\sqrt{a} \frac{x^{3/2}}{3/2} + \frac{x^2}{2} \right]_0^a$$

$$= a^2 - \frac{4}{3} a^{1/2} a^{3/2} + \frac{a^2}{2}$$

$$= a^2 - \frac{4}{3} a^2 + \frac{a^2}{2} = \frac{6a^2 - 8a^2 + 3a^2}{6} = \frac{a^2}{6} \text{ units}^2$$

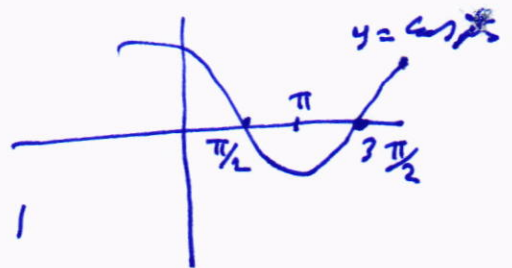
H.W

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. احمد أ. د. نجاد عبد الجليل

1 show that the area under $y = x^3$ over the interval $[0, b]$ is $b^4/4$

2 Find the area under the curve $y = 9 - x^2$ over the interval $[0, 3]$ ~~Ans~~ 18.

3 Find the area under the curve $y = \cos x$ over the interval $[0, \pi/2]$.



Ans: 1

4 Find the total area that is between the curve $y = x^2 - 3x - 10$, and the interval $[-3, 8]$.

Ans: $\frac{203}{2}$

5. Find the areas of the regions enclosed by the curves and lines

a/ $y = x$, $y = \frac{1}{x^2}$, $x = 2$, ans: 1

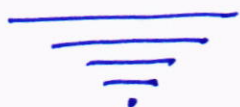
b/ $\sqrt{x} + \sqrt{y} = 1$, $x = 0$, $y = 0$, ans: $\frac{1}{6}$

c/ $x = 2y^2$, $x = 0$, $y = 3$, ans: 18

d/ $y = \sin x$, $y = x$, $0 \leq x \leq \frac{\pi}{4}$, ans: $\frac{\pi^2}{32} + \frac{\sqrt{2}}{2} - 1$

e/ $y^2 = 4x$, $y = 4x - 2$, ans: $\frac{9}{8}$

f/ $y = 2 \sin x$, $y = \sin 2x$, $0 \leq x \leq \pi$ ans: 4



VOLUMES

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. احمد أ. د. نجاد عبد الجليل

- 1 volumes by disks.
- 2 volumes by washers.
- 3 volumes by cylindrical shells:

1 volumes by disks.

$$V = \int_a^b \pi (f(x))^2 dx.$$

Ex: Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, and the lines $y = 1$, $x = 4$ about the line $y = 1$.

Sol

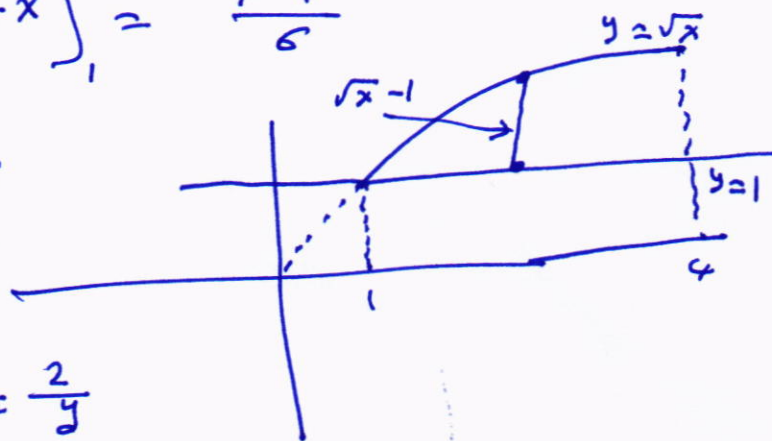
$$V = \int_a^b \pi (f(x))^2 dx$$

$$V = \int_1^4 \pi [\sqrt{x} - 1]^2 dx$$

$$V = \int_1^4 [x - 2\sqrt{x} + 1] dx$$

$$V = \pi \left[\frac{x^2}{2} - 2 \cdot \frac{2}{3} x^{3/2} + x \right]_1^4 = \frac{7\pi}{6}$$

Ex: Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = \frac{2}{y}$ $1 \leq y \leq 4$ about the y-axis.



Sol $V = \int_a^b (f(x))^2 dx$

$V = \int_1^4 \pi \left(\frac{2}{y}\right)^2 dy.$

$= \pi \int_1^4 \frac{4}{y^2} dy = 4\pi \left[-\frac{1}{y}\right]_1^4 = 4\pi \left[\frac{3}{4}\right] = 3\pi.$

Ex: Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$

Sol $y = -\sqrt{2}, y = +\sqrt{2}$

$V = \int_{-\sqrt{2}}^{\sqrt{2}} \pi [2 - y^2]^2 dy$

$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} [4 - 4y^2 + y^4] dy$

$= \pi \left[4y - \frac{4}{3}y^3 + \frac{y^5}{5} \right]_{-\sqrt{2}}^{\sqrt{2}}$

$= \frac{64\pi\sqrt{2}}{15}$

2 Volumes by washers:

$V = \int_a^b \pi \left[(f(x))^2 - (g(x))^2 \right] dx$

Ex: The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid, Find the volume of the solid.

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. اسد أ. د. شهاب الدين

Sol $x^2 + 1 = -x + 3$
 $x^2 + x - 2 = 0$
 $(x+2)(x-1) = 0$
 $x = -2, x = 1$

جامعة الزيتونة
 كلية الهندسة / المرحلة الأولى
 ص. ١٠٨ د. نزار عبد الجليل

$$V = \int_a^b \pi [(f(x))^2 - (g(x))^2] dx$$

$$V = \pi \int_{-2}^1 (-x+3)^2 - (x^2+1)^2 dx$$

$$V = \pi \int_{-2}^1 (8 - 6x - x^2 - x^4) dx$$

$$V = \pi \left[8x - 3x^2 - \frac{x^3}{3} - \frac{x^5}{5} \right]_{-2}^1 = \frac{117\pi}{5}$$

Ex: Find the volume of the solid, that is the region bounded by the parabola $y = x^2$ and the line $y = 2x$, revolved about the y-axis

Sol $f(y) = \sqrt{y}$, $g(y) = \frac{y}{2}$, $c = 0$, $d = 4$

$$V = \int_c^d \pi [(f(y))^2 - (g(y))^2] dy$$

$$= \int_0^4 \pi \left(\left[\sqrt{y} \right]^2 - \left[\frac{y}{2} \right]^2 \right) dy$$

$$= \pi \int_0^4 \left(y - \frac{y^2}{4} \right) dy = \pi \left[\frac{y^2}{2} - \frac{y^3}{12} \right]_0^4$$

$$= \frac{8}{3} \pi$$



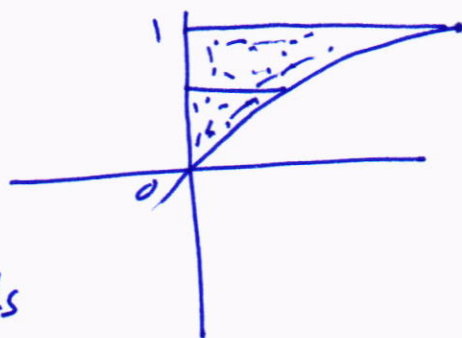
H.W

محاضر في الرياضيات
د. محمد عبد الحليم
الرياضة / المرحلة الأولى

- 1 Find the volume of the solid generated by revolving the shaded region about the y-axis

$$x = \tan\left(\frac{\pi}{4}\right) y$$

$$\text{ans: } 4 - \pi$$



- 2 Find the volumes of the solids generated by revolving the regions bounded by the lines and the curves in the following

a/ $y = x^2$, $y = 0$, $x = 2$ ans: $\frac{32\pi}{5}$

b/ $y = \sqrt{\cos x}$, $0 \leq x \leq \frac{\pi}{2}$, $y = 0$, $x = 0$ Ans π

c/ The region between the curve $y = \frac{1}{2\sqrt{x}}$ and the x-axis from $x = \frac{1}{4}$ to $x = 4$ Ans $\frac{\pi}{2} \ln 4$

- 3 Find the volumes of the solids generated by revolving the regions bounded by the lines and the curves in the following.

a/ The region enclosed by $x = \sqrt{5} y^2$, $x = 0$, $y = -1$, $y = 1$

$$\text{Ans } 2\pi$$

b/ $x = 2/\sqrt{y+1}$, $x = 0$, $y = 0$, $y = 3$ Ans: $4\pi \ln 4$

c/ The region enclosed by $x = \sqrt{2 \sin 2y}$, $0 \leq y \leq \frac{\pi}{2}$, $x = 0$

$$\text{Ans. } 2\pi$$

4 Find the volumes of the solids generated by revolving the regions bounded by the lines and curves.

حاصلات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. احمد ا. د. / لا داعي لطول

a $y = x$, $y = 1$, $x = 0$ Ans. $\frac{2\pi}{3}$

b $y = x^2 + 1$, $y = x + 3$ Ans: $\frac{117\pi}{5}$

c $y = \sec x$, $y = \sqrt{2}$, $\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$ Ans: $\pi(\pi - 2)$

5 Find the volume of the solid generated by revolving the following about y-axis

a The region enclosed by the triangle with the vertices $(1, 0)$, $(2, 1)$ and $(1, 1)$ Ans: $\frac{4\pi}{3}$

b The region bounded above by the parabola $y = x^2$ below by the x-axis, and on the right by the line $x = 2$. Ans: 8π

6: Find the volume of the solid generated, when the region between the curves of $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved about the x-axis.

Ans: $\frac{69\pi}{10}$



محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
مهنا ممداد أ. د. / دة دة دة دة

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
مهاسن د. أ. د. عبد الله

Ex: Use the cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x=1$, $x=4$ and the x -axis is revolved about the y -axis.

$\frac{3}{2} dx$

$$= \left[2\pi \cdot \frac{2}{5} x^{5/2} \right]_1^4 = \frac{4\pi}{5} [22-1] = \frac{124\pi}{5}$$

Ex: use cylindrical shells to find the volume of the solid generated, when the region enclosed by $y = x$, and $y = x^2$ is revolved about the y -axis.

A hand-drawn diagram of a coordinate system with a shaded sector in the first quadrant. The sector is bounded by the positive x-axis and a line segment. A small square indicates a right angle at the origin. The angle between the x-axis and the line segment is labeled as $\frac{1}{7} \pi$.

$$= 2\pi \int_0^1 (x^2 - x^3) dx$$

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1 = 2\pi \left[\frac{1}{3} - \frac{1}{4} \right] = \frac{\pi}{6}$$

Ex. Use cylindrical shell to find the volume of the solid generated when the region under $y=x^2$ over the interval $[0,2]$ is revolved about x -axis

محاضرات الرياضيات
كلية الهندسة / المرحلة الأولى
م. هادي أ. د. زياد عبد الجليل

Sol

x	y
0	0
2	4

$y = x^2 \Rightarrow x = \sqrt{y}$

$$V = \int_a^b 2\pi y [f(y) - g(y)] dy$$

$$V = \int_0^4 2\pi y [2 - \sqrt{y}] dy$$

$$V = 2\pi \int_0^4 [2y - y^{3/2}] dy = 2\pi \left[y^2 - \frac{y^{5/2}}{5/2} \right]_0^4 = \frac{32\pi}{5}$$

H.W.: 1) Use cylindrical shells to find the volumes of the solid generated when the region enclosed by the curves is revolved about y -axis.

a $y = x^3$, $x=1$, $y=0$ Ans: $\frac{2\pi}{5}$

b $x^2 + y^3 = 4$, $x=0$, $x=4$, $y=0$. Ans: $3\pi\sqrt[3]{4}(1+\sqrt[3]{3})$

c $y = 2x-1$, $y = -2x+3$, $x=2$ Ans: $\frac{2\pi}{3}$

2) Use cylindrical shells to find the volume of the solid generated when the region enclosed by the curve $y^2 = x$, $y=1$, $x=0$ is revolved about the x -axis. Ans: $\frac{\pi}{2}$

3 Use the shell method to find the volume of the solids generated by revolving the regions bounded by the curve $x = \sqrt{y}$, $x = -y$, $y=2$ about x -axis Ans: $\frac{16\pi}{15}(3\sqrt{2}+5)$

Arc Length

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. امداد أ. د. نجاد عبد الجليل

Length of a curve:

1. if $y = f(x)$, and $\frac{dy}{dx}$ is defined on $[a, b]$, then the arc length is

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

2. if $x = g(y)$, and $\frac{dx}{dy}$ is defined on $[c, d]$ then the arc length is:

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

Ex: Find the length of the curve

$$y = \frac{4\sqrt{2}}{3} x^{3/2} - 1, \quad 0 \leq x \leq 1$$

Sol $a = 0, \quad b = 1$
 $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1$

$$\frac{dy}{dx} = \frac{4\sqrt{2}}{3} \cdot \frac{3}{2} x^{1/2} = 2\sqrt{2} x^{1/2}$$

$$\left(\frac{dy}{dx}\right)^2 = \left[2\sqrt{2} x^{1/2}\right]^2 = 8x.$$

$$\begin{aligned} L &= \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^1 \sqrt{1 + 8x} dx \\ &= \frac{2}{3} \cdot \frac{1}{8} (1 + 8x)^{3/2} \Big|_0^1 = \frac{13}{6}. \end{aligned}$$

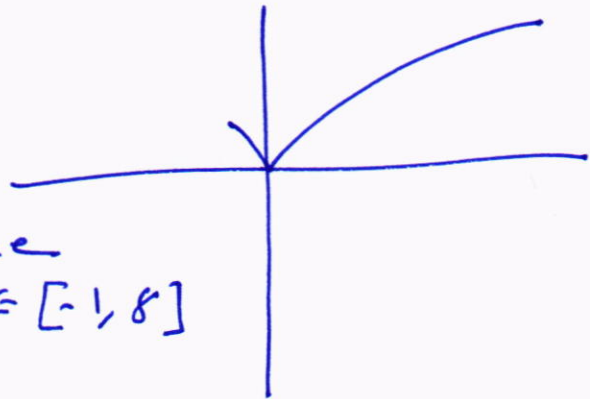
Ex Find the length of the graph $f(x) = \frac{x^3}{12} + \frac{1}{x}$, $1 \leq x \leq 4$

Sol $y' = \frac{x^2}{4} - \frac{1}{x^2}$

So:
$$L = \int_1^4 \sqrt{1 + (y')^2} dx$$
$$= \int_1^4 \left(\frac{x^2}{4} + \frac{1}{x^2} \right) dx$$
$$= \left[\frac{x^3}{12} - \frac{1}{x} \right]_1^4 = \left(\frac{64}{12} - \frac{1}{4} \right) - \left(\frac{1}{12} - 1 \right) = 6$$

Ex: Find the arc length of the curve $y = x^{\frac{2}{3}}$ between $x = -1$, and $x = 8$

Sol $y' = \frac{2}{3} x^{-\frac{1}{3}}$
 $= \frac{2}{3 x^{\frac{1}{3}}}$ is undefined at $x = 0 \in [-1, 8]$



$y = x^{\frac{2}{3}} \Rightarrow x = \pm y^{\frac{3}{2}}$

$L_1: x = -y^{\frac{3}{2}} \Rightarrow y' = -\frac{3}{2} y^{\frac{1}{2}}$, $0 \leq y \leq 1$

$L_1 = \int_0^1 \sqrt{1 + \frac{9}{4}y} dy = \frac{4}{9} \frac{(1 + \frac{9}{4}y)^{\frac{3}{2}}}{\frac{3}{2}} \Big|_0^1 = 1.74$

$L_2: x = y^{\frac{3}{2}} \Rightarrow \frac{dx}{dy} = \frac{3}{2} y^{\frac{1}{2}}$, $0 \leq y \leq 4$

$L_2 = \int_0^4 \sqrt{1 + \frac{9}{4}y} dy = \frac{4}{9} \frac{(1 + \frac{9}{4}y)^{\frac{3}{2}}}{\frac{3}{2}} \Big|_0^4$

$L = L_1 + L_2$

H.W

1 Find the length of the curves

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. احمد أ. خ. محمد عبد الله

a $y = \left(\frac{1}{3}\right) (x^2 + 2)^{3/2}$ from $x=0$ to $x=3$: ans 12

b $y = \left(\frac{3}{4}\right) x^{4/3} - \left(\frac{3}{8}\right) x^{2/3} + 5$, $1 \leq x \leq 8$ Ans: $\frac{99}{8}$

2 Find the arc length of the curve $y = x^{3/2}$
from $(1,1)$ to $(2, 2\sqrt{2})$. Ans $\frac{22\sqrt{22} - 13\sqrt{13}}{27}$

3 Find the arc length of the curve $y = 2x$ from
 $(1,2)$ to $(2,4)$ Ans $\frac{1}{243} (85\sqrt{85} - 8)$

4 Find the arc length of the curve :
 $24xy = y^4 + 48$ from $y=2$ to $y=4$

Ans: $\frac{17}{6}$



Inverse Function:

if $f: A \rightarrow B$ then $f^{-1}: B \rightarrow A$

or if $y = f(x)$, then $x = f^{-1}(y)$

$$D_{f^{-1}} = R_f \quad \text{and} \quad R_{f^{-1}} = D_f$$

Ex: Let $f(x) = \frac{x}{x-2}$, Find $f^{-1}(x)$

Sol $y = \frac{x}{x-2} \Rightarrow x = yx - 2y \Rightarrow x - yx = -2y$

$$x(1-y) = -2y \Rightarrow x = \frac{-2y}{1-y}$$

Put x by y we obtain. $\Rightarrow y = \frac{-2x}{1-x}$

$$y = \frac{-2x}{1-x} = f^{-1}(x).$$

Note: if $f(x)$ is increase or decrease always. Then f has an inverse.

Ex: Show that $f(x) = x^5 + 7x^3 + 4x + 1$ has an inverse.

$$f'(x) = 5x^4 + 21x^2 + 4 > 0 \quad \text{always increase thus}$$

$f(x)$ has an inverse.

Note: $f(x) \circ f^{-1}(x) = f^{-1}(x) \circ f(x) = x$.

Ex: Determine whether $f(x) = \sqrt[3]{x-2}$ is the increase
of $g(x) = x^3 + 2$

Sol

$$f \circ g = f(g(x))$$

$$= f(x^3 + 2)$$

$$= \sqrt[3]{(x^3 + 2) - 2}$$

$$= \sqrt[3]{x^3}$$

$$= x$$

$$\therefore (f \circ g)_x = x$$

$$\therefore g(x) = f^{-1}(x)$$

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. احمد أ. د. محمد عبد الجليل

Logarithms:

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
مهاomed أ. د. نجاد عبد الجليل

Def: $\text{Log}_a x = y$
 $\Leftrightarrow x = a^y$

Properties:

1. $\text{Log}_a (x \cdot y) = \text{Log}_a x + \text{Log}_a y$
2. $\text{Log}_a \left(\frac{x}{y}\right) = \text{Log}_a x - \text{Log}_a y$
3. $\text{Log}_a x^n = n \text{Log}_a x$
4. $\text{Log}_a a = 1, \text{Log}_a 1 = 0$

Ex: $\text{Log}_{10} 100 = 2$, Since $10^2 = 100$
 $\text{Log}_2 8 = 3$, Since $2^3 = 8$
 $\text{Log}_{10} \frac{1}{1000} = -3$, Since $10^{-3} = \frac{1}{1000}$

Theorem:

$\text{Log}_b e = \text{Log}_e e = 1$ put $b = e$, $e = 2.71828 \dots$

Ex: Solve for x :
 $\text{Log}_5 (5^{2x}) = 8$

Sol $2x \text{Log}_5 5 = 8$
 $2x \cdot 1 = 8$
 $\therefore x = 4$

Ex: Solve $\text{Log}_{10} x^2 + \text{Log}_{10} x = 30$

Sol $2 \text{Log}_{10} x + \text{Log}_{10} x = 30$
 $3 \text{Log}_{10} x = 30 \Rightarrow \text{Log}_{10} x = 10 \Rightarrow x = 10^{10}$

Note: if $y = \log_a u$ then:

$$\frac{dy}{dx} = \frac{1}{u} \log_a e \cdot \frac{du}{dx} \quad \text{where } e \approx 2.7$$

محاضرات في الرياضيات
للمرحلة الهندسة / المرحلة الأولى
م. ابراهيم أ. د. في الرياضيات

Ex Find y' , if $\log_2 x$

$$\frac{dy}{dx} = \frac{1}{x} \log_2 e.$$

Ex, Find y' , if $y = \log_3 (2x^2 + 5x + 1)$

$$\frac{dy}{dx} = \frac{1}{2x^2 + 5x + 1} \log_3 e \cdot (4x + 5)$$

$$= \frac{4x + 5}{2x^2 + 5x + 1} \log_3 e.$$

Note: $b^{\log_b x} = x$ for $x > 0$.

Natural Logarithm:

if $a = e$ Then $\log_a x = \log_e x = \ln x$

$$y = f(x) = \ln x.$$

$$D_f = \{x : x > 0\}.$$

$$R_f = \{R\}.$$

Ex: Find the domain of $f(x) = \log_3 (x - 2)$

Sol

$$x - 2 > 0$$

$$x > 2$$

$$\therefore P_f = \{x : x > 2\}.$$

Theorem: For any positive numbers a and c and any rational number r .

محاضرات في الرياضيات
م. ا. د. ن. د. عبد الباقى
كلية الطب - المرحلة الأولى :

a $\ln 1 = 0$

b $\ln ac = \ln a + \ln c$

c $\ln \frac{a}{c} = \ln a - \ln c$

d $\ln a^r = r \ln a$

e $\ln \frac{1}{c} = -\ln c$

Ex: Draw the graph of $y = \ln\left(\frac{1}{x}\right)$

Sol $\ln\left(\frac{1}{x}\right) = \ln 1 - \ln x$
 $= 0 - \ln x$
 $= -\ln x$

$\therefore \ln\left(\frac{1}{x}\right) = -\ln x$

Ex: Draw the graph of $y = \ln|x|$

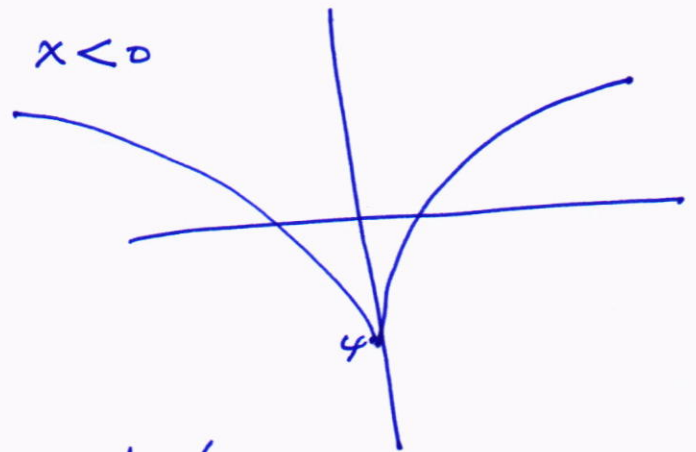
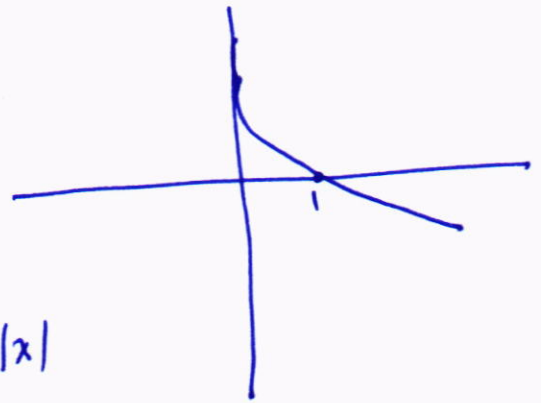
Sol $y = \ln|x| = \begin{cases} \ln x, & x \geq 0 \\ \ln(-x), & x < 0 \end{cases}$

if $y = \ln u$ then :

$$\frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$$

Ex: Let $y = \ln(\sin x)$ Find y'

Sol $\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$



Ex: Find $\frac{dy}{dx}$, if $y = \ln\left(\frac{x^2 \sin^3 x}{\cos x^4 \sqrt{1+x}}\right)$

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. اسد أ. د. نجاد عبد الجليل

Sol $y = \ln\left(\frac{x^2 \sin^3 x}{\cos x^4 \sqrt{1+x}}\right)$

$$y = \ln(x^2 \sin^3 x) - \ln(\cos x^4 \sqrt{1+x})$$

$$y = \ln x^2 + \ln \sin^3 x - [\ln \cos x^4 + \ln(1+x)^{\frac{1}{2}}]$$

$$y = 2 \ln x + 3 \ln \sin x - \ln \cos x^4 - \frac{1}{2} \ln(1+x)$$

$$y' = 2 \cdot \frac{1}{x} + 3 \cdot \frac{1}{\sin x} \cos x - \frac{1}{\cos x^4} (-\sin x^4 \cdot (4x^3)) - \frac{1}{2} \cdot \frac{1}{1+x}$$

$$y' = \frac{2}{x} + 3 \cot x + 4x^3 \cot x^2 - \frac{1}{2(1+x)}$$

Ex: Let $y = x^{\sin x}$, Find y' .

Sol $y = x^{\sin x} \Rightarrow \ln y = \ln x^{\sin x}$

$$\ln y = \sin x \ln x$$

$$\frac{1}{y} \cdot y' = \sin x \frac{1}{x} + \ln x \cdot \cos x$$

$$\therefore y' = y \left[\frac{\sin x}{x} + \ln x \cos x \right] = x^{\sin x} \left[\frac{\sin x}{x} + \ln x \cos x \right]$$

Ex: Find y' , if $y = \frac{x^3 \sqrt{x-5}}{\sin^2 x \cos^3 x}$

Sol: $\ln y = \ln\left(\frac{x^3 \sqrt{x-5}}{\sin^2 x \cos^3 x}\right)$

$$\ln y = \ln[x^3 \sqrt{x-5}] - [\ln \sin^2 x + \ln \cos^3 x]$$

$$\ln y = 3 \ln x + \frac{1}{2} \ln(x-5) - 2 \ln \sin x - 3 \ln \cos x$$

$$\frac{1}{y} \cdot y' = 3 \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{x-5} - 2 \frac{1}{\sin x} \cos x - 3 \frac{1}{\cos x} (-\sin x)$$

$$y' = y \left[\frac{3}{x} + \frac{1}{2(x-5)} - 2 \cot x + 3 \tan x \right]$$

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
مه أمداد أ. د. نوري عبد الجليل

$$\therefore y' = \frac{x^3 \sqrt{x-5}}{\sin^2 x \cos^3 x} \left[\frac{3}{x} + \frac{1}{2(x-5)} - 2 \cot x + 3 \tan x \right]$$

$$\int \frac{du}{u} = \ln|u| + C.$$

Ex: $\int \frac{x^2 dx}{x^3 - 4} = \frac{1}{3} \int \frac{3x^2 dx}{x^3 - 4} = \frac{1}{3} \ln|x^3 - 4| + C.$

Ex: Find $\int \frac{1}{\sqrt{x}(1+\sqrt{x})} dx$

Sol let $u = 1 + \sqrt{x}$
 $du = \frac{1}{2\sqrt{x}} dx \Rightarrow 2 du = \frac{dx}{\sqrt{x}}$
 $= \int \frac{2 du}{u} = 2 \int \frac{du}{u} = 2 \ln|u| + C$
 $= 2 \ln|1 + \sqrt{x}| + C.$

Ex: Find $\int \frac{\cos(\ln x)}{x} dx.$

Sol let $u = \ln x \Rightarrow du = \frac{1}{x} \cdot dx$
 $= \int \cos u du = \sin u + C$
 $= \sin(\ln x) + C$

Ex: $\int \frac{\cot x dx}{1} = \int \frac{\cos x}{\sin x} dx = \ln|\sin x| + C$

Ex: $\int \sec x dx = \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x}$
 $= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx = \ln|\sec x + \tan x| + C$

Ex: $\int \frac{\ln x}{x} dx$, let $u = \ln x$

$\therefore du = \frac{1}{x} dx$

$= \int u du = \frac{u^2}{2} + c = \frac{(\ln x)^2}{2} + c$

Ex: $\int \frac{dx}{x \ln x}$

Sol let $u = \ln x \Rightarrow du = \frac{1}{x} \cdot dx$

$= \int \frac{du}{u} = \ln |u| + c = \ln |\ln x| + c$

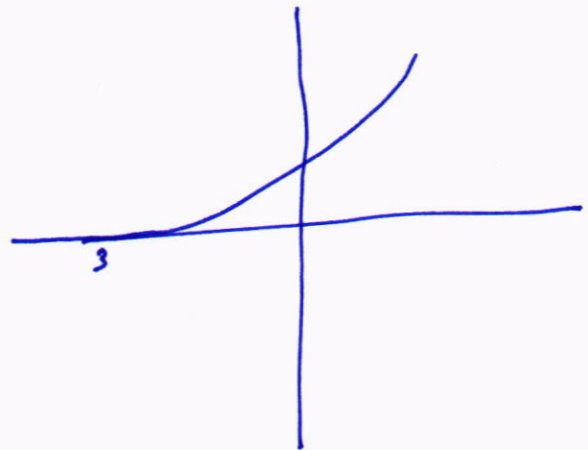
Exponential Function:

$y = f(x) = e^x$

$D_f = \mathbb{R}$

$R_f = \{y, y > 0\}$

$\lim_{x \rightarrow \infty} e^x = \infty, \lim_{x \rightarrow -\infty} e^x = 0$



properties,

1. $e^x \cdot e^y = e^{x+y}$

2. $\frac{e^x}{e^y} = e^{x-y}$

3. $(e^x)^n = e^{nx}$

Note $\ln e = 1$

if $y = e^u$ then $y' = e^u \cdot \frac{du}{dx}$

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
مهاجر أ. د. عبد الجليل

Ex: Find y' , if $y = e^{x \tan x}$.

Sol $y' = e^{x \tan x} \cdot [x \sec^2 x + \tan x \cdot 1]$

$\int e^u du = e^u + C$

Ex: $\int \frac{e^{\tan x}}{\cos^2 x} dx$

Sol $\int e^{\tan x} \sec^2 x dx$

let $u = \tan x$, $du = \sec^2 x dx$

$= \int e^u du = e^u + C = e^{\tan x} + C$

Ex: $\int e^{x^2} dx = \int e^{\ln x^2} dx = \int x^2 dx = \frac{x^3}{3} + C$

Ex: $\int e^{x+e^x} dx = \int e^x \cdot e^x dx$, let $u = e^x \Rightarrow du = e^x dx$
 $= \int e^u du = e^u + C$
 $= e^{e^x} + C$

if $y = f(x) = a^x$, $a > 0$

1. $a^x \cdot a^y = a^{x+y}$

2. $\frac{a^x}{a^y} = a^{x-y}$

3. $(a^x)^n = a^{nx}$

$$\boxed{\text{if } y = a^u, \text{ then } y' = a^u \cdot \frac{du}{dx} \cdot \ln a}$$

محاضرات في الرياضيات
كلية الطب / المرحلة الأولى
مهاجر أ. د. نزيه دويش

Ex: Find y' , if $y = \frac{\sin x}{2}$

Sol: $\frac{dy}{dx} = \frac{\sin x}{2} \cdot \cos x \cdot \ln 2.$

Ex: Find y' , if $y = \pi^{x^2} + e^x \cdot x^e.$

Sol: $y' = \pi^{x^2} \cdot (2x) \cdot \ln \pi + [e^x \cdot e^{x-1} + x^e \cdot e^x]$

$$\boxed{\int a^u du = \frac{1}{\ln a} \cdot a^u + c}$$

Ex: $\int \frac{\sin x}{2} \cos x dx$, Let $u = \sin x \Rightarrow du = \cos x dx$

$$= \int 2^u du = \frac{1}{\ln 2} \cdot 2^u + c$$

$$= \frac{1}{\ln 2} \cdot \frac{\sin x}{2} + c.$$

Ex: Find $\lim_{x \rightarrow \infty} \frac{2+e^x}{1+3e^x}$, note $e^\infty = \infty$, $e^{-\infty} = 0$

Sol: $\lim_{x \rightarrow \infty} \frac{2+e^x}{1+3e^x} \cdot \frac{e^{-x}}{e^{-x}}$

$$= \lim_{x \rightarrow \infty} \frac{2e^{-x} + 1}{e^{-x} + 3} = \frac{0+1}{0+3} = \frac{1}{3}$$



The hyperbolic Functions

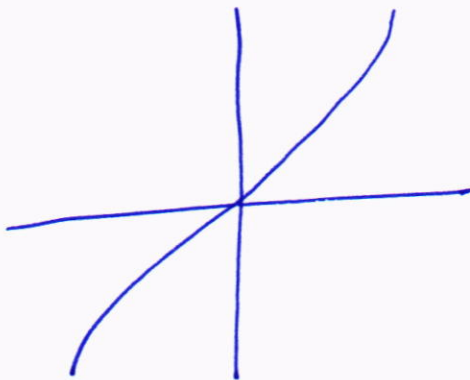
محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. اعداد أ. د. ن. د. عبد الجليل

Definition:

The hyperbolic Sine and hyperbolic cosine functions, denoted by \sinh and \cosh , respectively, are defined by

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

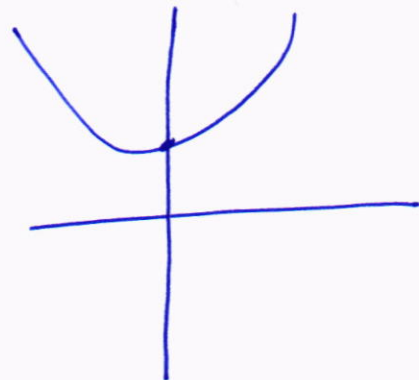
$$\cosh x = \frac{e^x + e^{-x}}{2}$$



$$y = \sinh x$$

$$f(x) = \sinh x$$

$$D_f = R_f = R$$



$$y = \cosh x$$

$$f(x) = \cosh x$$

$$D_f = R$$

$$R_f = [1, \infty)$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

1. $\cosh^2 x - \sinh^2 x = 1$
2. $1 - \tanh^2 x = \operatorname{sech}^2 x$
3. $\coth^2 x - 1 = \operatorname{csch}^2 x$
4. $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y.$
5. $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y.$
6. $\cosh x + \sinh x = e^x$
7. $\cosh x - \sinh x = e^{-x}$
8. $\sinh 2x = 2 \sinh x \cosh x$
9. $\cosh 2x = \begin{cases} \cosh^2 x + \sinh^2 x \\ 2 \sinh^2 x + 1 \\ 2 \cosh^2 x - 1 \end{cases}$
10. $\cosh(-x) = \cosh x$
11. $\sinh(-x) = -\sinh x.$

Derivative	Integrals
1. $(\sinh x)' = \cosh x$	1. $\int \cosh x dx = \sinh x + c$
2. $(\cosh x)' = \sinh x$	2. $\int \sinh x dx = \cosh x + c$
3. $(\tanh x)' = \operatorname{sech}^2 x$	3. $\int \operatorname{sech}^2 x dx = \tanh x + c$
4. $(\coth x)' = -\operatorname{csch}^2 x$	4. $\int \operatorname{csch}^2 x dx = -\coth x + c$
5. $(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$	5. $\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + c$
6. $(\operatorname{csch} x)' = -\operatorname{csch} x \coth x$	6. $\int \operatorname{csch} x \coth x dx = -\operatorname{csch} x + c$

Ex: prove that.

$$\cosh(3x) = 4 \cosh^3 x - 3 \cosh x$$

Sol $\cosh 3x = \cosh(x+2x)$

$$= \cosh x \cosh 2x + \sinh x \sinh 2x$$

$$= \cosh x (2 \cosh^2 x - 1) + \sinh x (2 \sinh x \cosh x)$$

$$= 2 \cosh^3 x - \cosh x + 2 \sinh^2 x \cosh x$$

$$= 2 \cosh^3 x - \cosh x + 2(\cosh^2 x - 1) \cosh x$$

$$= 2 \cosh^3 x - \cosh x + 2 \cosh^3 x - 2 \cosh x$$

$$= 4 \cosh^3 x - 3 \cosh x$$

Ex: Solve $\sinh x = \frac{1}{4} \cosh x$ in terms of $\ln 3, \ln 5$.

Sol $\sinh x = \frac{1}{4} \cosh x$

$$\frac{e^x - e^{-x}}{2} = \frac{1}{4} \cdot \frac{e^x + e^{-x}}{2}$$

$$\left[\frac{e^x - e^{-x}}{2} = \frac{1}{4} \cdot \frac{e^x + e^{-x}}{2} \right] \times 2$$

$$\left[e^x - e^{-x} = \frac{1}{4} e^x + \frac{1}{4} e^{-x} \right] \times e^x$$

$$e^{2x} - 1 = \frac{1}{4} e^{2x} + \frac{1}{4}$$

$$e^{2x} - \frac{1}{4} e^{2x} = 1 + \frac{1}{4}$$

$$\left[\frac{3}{4} e^{2x} = \frac{5}{4} \right] \times \frac{4}{3}$$

$$e^{2x} = \frac{5}{3} \Rightarrow 2x = \ln \frac{5}{3}$$

$$x = \frac{1}{2} [\ln 5 - \ln 3]$$



Ex: Find $\frac{d}{dx} \left[\int_0^{\ln x} \frac{dt}{\sqrt{4+e^t}} \right]$

محاضرات في الرياضيات
كلية الطب / المرحلة الأولى
مادة: حساب التفاضل والتكامل

Sol By The second Fundamental

Theorem of Integral: which say that

if $F(x) = \int_a^x f(t) dt$, then $F'(x) = f(x)$

$t = \ln x \Rightarrow dt = \frac{1}{x}, e^{\ln x} =$

$= \frac{1}{\sqrt{4+e^{\ln x}}} \cdot \frac{1}{x} = \frac{1}{x\sqrt{4+x}}$

Ex: show that $y = e^{ax} \sinh bx$ satisfies

$y'' - 2ay' + (a^2 + b^2)y = 0.$

Sol $y' = b e^{ax} \cosh bx + a e^{ax} \sinh bx.$

$y'' = b^2 e^{ax} \sinh bx + ab e^{ax} \cosh bx + ab e^{ax} \cosh bx + a^2 e^{ax} \sinh bx$

$y'' = e^{ax} \sinh bx (a^2 + b^2) + 2ab e^{ax} \cosh bx$

$y'' - 2ay' + (a^2 + b^2)y = (a^2 + b^2)e^{ax} \sinh bx + 2ab e^{ax} \cosh bx$

$- 2a [b e^{ax} \cosh bx + a e^{ax} \sinh bx] + (a^2 + b^2) e^{ax} \sinh bx = 0.$

Ex: Evaluate: $\int [\tanh x + \sqrt{\tanh x} \cdot \operatorname{sech}^2 x] dx$
 $= \int \frac{\sinh x}{\cosh x} dx + \int (\tanh x)^{1/2} \operatorname{sech}^2 x dx$
 $= \ln |\cosh x| + \frac{(\tanh x)^{3/2}}{3/2} + C$

Ex: Evaluate: $\int \frac{\sinh 2x dx}{3 + 5 \cosh 2x}$

Let $u = 3 + 5 \cosh 2x \Rightarrow du = 10 \sinh 2x dx$

$$\int \frac{\sinh 2x dx}{3 + 5 \cosh 2x}$$

$$= \frac{1}{10} \int \frac{du}{u} = \frac{1}{10} \ln |u| + C$$

$$= \frac{1}{10} \ln |3 + 5 \cosh 2x| + C.$$

Ex: show that: $\lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = 1$

$$\begin{aligned} \text{Sol } \lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} &= \lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot \frac{e^x}{e^x} \\ &= \lim_{x \rightarrow \infty} \frac{e^{2x} - 1}{e^{2x} + 1} = 1 \end{aligned}$$

Inverse trigonometric and hyperbolic Functions.

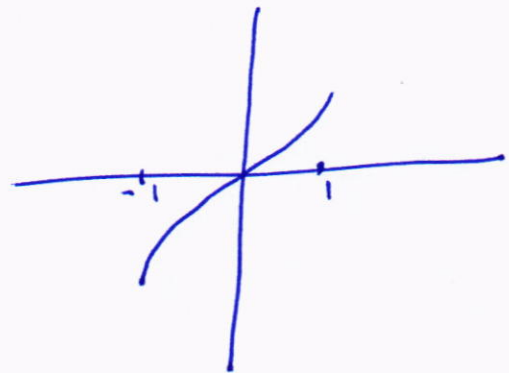
if $-1 \leq x \leq 1$, and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, then $y = \sin^{-1} x$ and $\sin y = x$
 $\sin^{-1}(\sin y) = y$, if $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$\sin(\sin^{-1} x) = x$, if $-1 \leq x \leq 1$

$$1) f(x) = \sin^{-1} x$$

$$D_f = [-1, 1]$$

$$R_f = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$



Ex: Simplify:

$$\cos(\sin^{-1} x)$$

$$\text{Sol let } \alpha = \sin^{-1} x \Rightarrow \frac{x}{1} = \sin \alpha$$

$$\therefore \cos(\sin^{-1} x) = \cos \alpha = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}.$$

Ex: Simplify:

$$\sec^2(\tan^{-1} x)$$

Sol let $\alpha = \tan^{-1} x \Rightarrow \tan \alpha = \frac{x}{1}$

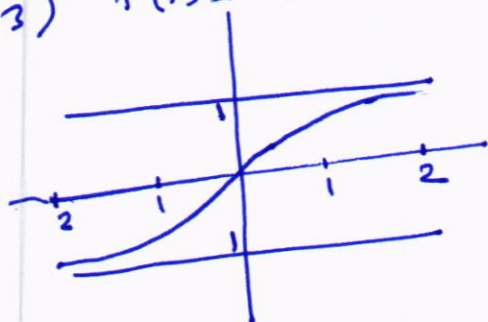
$$\therefore \sec(\tan^{-1} x) = \sec \alpha = \frac{\sqrt{1+x^2}}{1} = \sqrt{1+x^2}$$

$$= \sec^2(\tan^{-1} x) = 1+x^2.$$

2) $f(x) = \cos^{-1} x$

$$D_f = [-1, 1], R_f = [0, \pi]$$

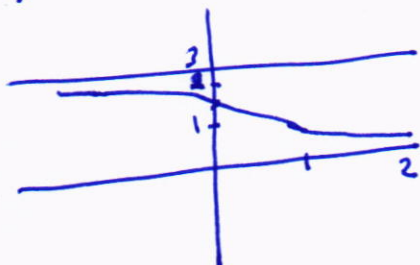
3) $f(x) = \tan^{-1} x$



$$D_f = \mathbb{R}$$

$$R_f = (-\frac{\pi}{2}, \frac{\pi}{2})$$

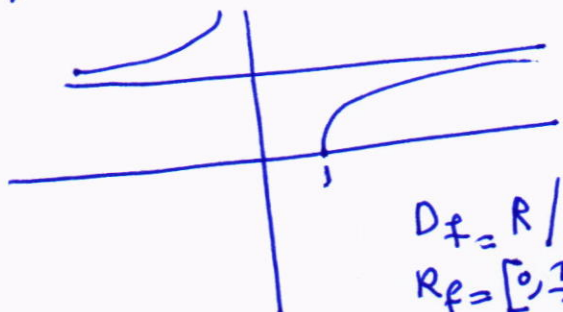
4) $f(x) = \cot^{-1} x$



$$D_f = \mathbb{R}$$

$$R_f = (0, \pi).$$

5) $f(x) = \sec^{-1} x$



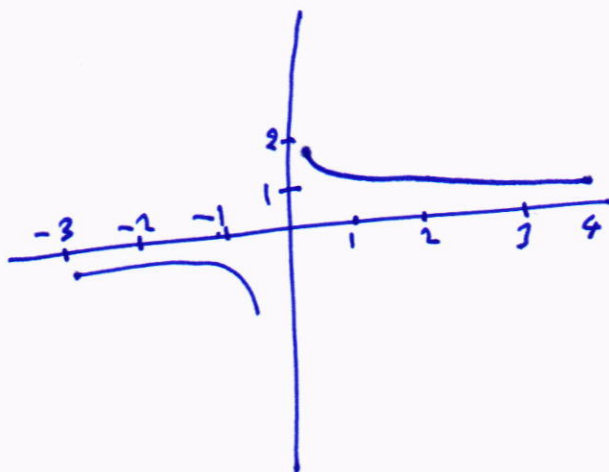
$$D_f = \mathbb{R} / (-1, 1)$$

$$R_f = [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$$

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. اعداد أ. د. نورا عبد الجليل



6) $f(x) = \csc^{-1} x$



$$D_f = \mathbb{R} / (-1, 1)$$

$$R_f = [-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}].$$

Ex: Prove that:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

مأذات في الرياضيات
كلية الهندسة / المرحلة الأولى
مهاجد أ.د. زياد عبد الجليل

Sol let $y = \cos^{-1} x \Rightarrow x = \cos y$

$$x = \sin\left(\frac{\pi}{2} - y\right)$$

$$\sin^{-1} x = \frac{\pi}{2} - y$$

$$y = \frac{\pi}{2} - \sin^{-1} x \quad \therefore \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$\text{or } \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

Ex: Find the value of $\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right)$.

Sol let $\alpha = \cos^{-1} \frac{3}{5} \Leftrightarrow \cos \alpha = \frac{3}{5}$

$$\therefore \sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right) = \sin 2\alpha$$

$$= 2 \sin \alpha \cos \alpha$$

$$= 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25}$$

Ex: Find the value of $\sin\left[\sin^{-1}\left(\frac{2}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right)\right]$

Sol: let $\alpha = \sin^{-1} \frac{2}{3}$, $\beta = \cos^{-1}\left(\frac{1}{3}\right)$.

$$\Downarrow \quad \Downarrow$$
$$\sin \alpha = \frac{2}{3}, \quad \cos \beta = \frac{1}{3}$$

$$\sin[\alpha + \beta] = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$= \frac{2}{3} \cdot \frac{1}{3} + \frac{\sqrt{5}}{3} \cdot \frac{\sqrt{8}}{3}$$

$$= \frac{2}{9} + \frac{\sqrt{40}}{9} = \frac{2 + \sqrt{40}}{9}$$

Ex: Prove that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

Sol let $\alpha = \tan^{-1} x \Rightarrow \tan \alpha = x$

$$\beta = \tan^{-1} y \Rightarrow \tan \beta = y$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\therefore \alpha + \beta = \tan^{-1} \left(\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \right)$$

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. احمد ا. د. نزار عبد الجليل

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$$

Derivatives and Integrals of inverse trigonometric Functions

Theorem:

1. $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$
2. $(\cos^{-1} x)' = -\frac{1}{\sqrt{1-x^2}}$
3. $(\tan^{-1} x)' = \frac{1}{1+x^2}$
4. $(\cot^{-1} x)' = -\frac{1}{1+x^2}$
5. $(\sec^{-1} x)' = \frac{1}{x\sqrt{x^2-1}}$
6. $(\csc^{-1} x)' = \frac{1}{x\sqrt{x^2-1}}$

Integrals

1. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$
2. $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$
3. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c.$

Ex: Find $\frac{dy}{dx}$, if $y = \sin^{-1}(x^3)$.

Sol:

$$\begin{aligned} y' &= \frac{1}{\sqrt{1-(x^3)^2}} \cdot (3x^2) \\ &= \frac{3x^2}{\sqrt{1-x^6}} \end{aligned}$$

Ex: Find $\frac{dy}{dx}$, if $y = \sec^{-1} e^x$.

Sol:

$$y' = \frac{1}{e^x \sqrt{(e^x)^2 - 1}} e^x = \frac{1}{\sqrt{e^{2x} - 1}}$$

Ex: Evaluate.

$$\int \frac{dx}{1+3x^2}$$

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
مهنا د. أ. د. نزار عبد الجليل

Sol Substituting:

$$u = \sqrt{3} x, \quad du = \sqrt{3} \cdot dx$$

$$\Rightarrow \int \frac{dx}{1+3x^2} = \frac{1}{\sqrt{3}} \int \frac{du}{1+u^2} = \frac{1}{\sqrt{3}} \tan^{-1} u + c \\ = \frac{1}{\sqrt{3}} \tan^{-1}(\sqrt{3} x) + c.$$

Ex: Evaluate:

$$\int \frac{e^x}{\sqrt{1-e^{2x}}} dx$$

Sol: let: $u = e^x, \quad du = e^x \cdot dx.$

$$\Rightarrow \int \frac{e^x}{\sqrt{1-e^{2x}}} dx = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + c = \sin^{-1}(e^x) + c$$

Ex Evaluate: $\int \frac{dx}{a^2+x^2}$, where $a \neq 0$ is a constant

Sol let $x = au \Rightarrow dx = a du.$

$$\text{then } \int \frac{dx}{a^2+x^2} = \int \frac{a du}{a^2+a^2 u^2} = \frac{1}{a} \int \frac{du}{1+u^2}$$

$$= \frac{1}{a} \tan^{-1} u + c = \frac{1}{a} \tan^{-1} \frac{x}{a} + c.$$

Ex: Evaluate:

$$\int \frac{dx}{\sqrt{2-x^2}}, \text{ By Method of above example.}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c, \quad a > 0. \\ \therefore \int \frac{dx}{\sqrt{2-x^2}} = \sin^{-1} \frac{x}{\sqrt{2}} + c$$

Ex:

$$\int \frac{dx}{\sqrt{9-x^2}}$$

جامعة الزيتونة
كلية الهندسة / المرحلة الأولى
م. اسد أ. د. نجاد عبد الحليم

Sol

$$\int \frac{dx}{\sqrt{9(1-\frac{x^2}{9})}} = \int \frac{dx}{3\sqrt{1-(\frac{x}{3})^2}}$$

$$\text{let } u = \frac{x}{3} \Rightarrow du = \frac{1}{3} dx$$

$$= \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + c = \sin^{-1} \left(\frac{x}{3}\right) + c.$$

Inverse of Hyperbolic Functions

Ex prove that: $\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$

Sol let $y = \sinh^{-1} x \Rightarrow \sinh y = x$

$$\frac{e^y - e^{-y}}{2} = x$$

$$[e^y - e^{-y} = 2x] \times e^y$$

$$e^{2y} - 1 = 2x e^y$$

$$1. e^{2y} - 2x e^y - 1 = 0$$

$$a=1, \quad b=-2x, \quad c=-1$$

$$\therefore e^y = \frac{2x \pm \sqrt{4x^2+4}}{2}$$

$$e^y = \frac{2x + 2\sqrt{x^2+1}}{2}$$

$$e^y = x + \sqrt{x^2+1}, \quad \text{since } e^y > 0$$

$$e^y = x + \sqrt{x^2+1}$$

$$y = \ln(x + \sqrt{x^2+1})$$

$$\therefore \sinh^{-1} x = \ln(x + \sqrt{x^2+1})$$

Ex: Find $\frac{dy}{dx}$ of

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
م. ابراهيم أ. د. د. عبد المطلب

1) $y = 2^x + \sinh^{-1}(x^2)$

$$\frac{dy}{dx} = 2^x \cdot \ln 2 + \frac{1}{\sqrt{1+x^4}} \cdot 2x$$

2) $y = (1 + x \csc h^{-1} x)^{10}$

$$y' = 10(1 + x \csc h^{-1} x)^9 \cdot \left[x \cdot \frac{-1}{|x|\sqrt{1+x^2}} + \csc h^{-1} x \cdot 1 \right]$$

Ex: show that if $y = \tan^{-1} x$ then $y'' = -2 \sin y \cos^3 y$

Sol $y = \tan^{-1} x \Rightarrow \tan y = x$

$$\sec^2 y \cdot y' = 1$$

or $y' = \frac{1}{\sec^2 y} = \cos^2 y$

or $y' = [\cos y]^2$

$$y'' = 2[\cos y] - (-\sin y) y'$$

$$y'' = 2[\cos y] - (-\sin y)(\cos^2 y)$$

$$\therefore y'' = -2 \sin y \cos^3 y.$$



Techniques of Integration

مفاتيح في الرياضيات
كلية الهندسة / المرحلة الأولى
م. اسرار أ. د. محمد عبد الجليل

Trigonometric Substitutions:

1. if $\sqrt{a^2 - u^2}$ then let $u = a \sin \theta$.
2. if $\sqrt{a^2 + u^2}$ then let $u = a \tan \theta$.
3. if $\sqrt{u^2 - a^2}$ then let $u = a \sec \theta$.

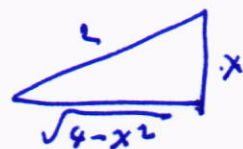
Ex: Evaluate: $\int \frac{dx}{x^2 \sqrt{4-x^2}}$

Sol: let $x = 2 \sin \theta \Rightarrow dx = 2 \cos \theta d\theta$.

$$\begin{aligned}
 &= \int \frac{2 \cos \theta d\theta}{4 \sin^2 \theta \sqrt{4-4 \sin^2 \theta}} \\
 &= \frac{1}{2} \int \frac{\cos \theta d\theta}{\sin^2 \theta \cdot 2 \sqrt{1-\sin^2 \theta}} \\
 &= \frac{1}{4} \int \frac{\cos \theta d\theta}{\sin^2 \theta \sqrt{\cos^2 \theta}} = \frac{1}{4} \int \frac{\cos \theta d\theta}{\sin^2 \theta \cos \theta} \\
 &= \frac{1}{4} \int \csc^2 \theta d\theta \\
 &= -\frac{1}{4} \cot \theta + C.
 \end{aligned}$$

$\therefore x = 2 \sin \theta \Rightarrow \frac{x}{2} = \sin \theta$

$= -\frac{1}{4} \frac{\sqrt{4-x^2}}{x} + C.$



Ex: Evaluate: $\int \frac{dx}{(3+x^2)^{3/2}}$

Sol let $x = \sqrt{3} \tan \theta \Rightarrow dx = \sqrt{3} \sec^2 \theta d\theta$

$\frac{x}{\sqrt{3}} = \tan \theta$

$= \int \frac{\sqrt{3} \sec^2 \theta d\theta}{(3+3 \tan^2 \theta)^{3/2}}$

$$= \sqrt{3} \int \frac{\sec^2 \theta d\theta}{\frac{3}{2} (1 + \tan^2 \theta)^{3/2}}$$

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
مساعد أ.د. نزيه عبد المطلب

$$= \frac{1}{3} \int \frac{\sec^2 \theta d\theta}{(\sec^2 \theta)^{3/2}} = \frac{1}{3} \int \frac{\sec^2 \theta d\theta}{\sec^3 \theta}$$

$$= \frac{1}{3} \int \frac{d\theta}{\sec \theta} = \frac{1}{3} \int \cos \theta d\theta = \frac{1}{3} \sin \theta + C$$

$$= \frac{1}{3} \frac{x}{\sqrt{3+x^2}} + C.$$

Integration involving ax^2+bx+c

Ex: Evaluate:

$$\int \frac{dx}{\sqrt{5-4x-2x^2}}$$

Sol

$$5-4x-2x^2 = -2(x^2+2x) + 5$$

$$= -2(x^2+2x+1-1) + 5$$

$$= -2[(x+1)^2 - 1] + 5$$

$$= -2(x+1)^2 + 2 + 5$$

$$= 7 - 2(x+1)^2$$

$$\int \frac{dx}{\sqrt{5-4x-2x^2}} = \int \frac{dx}{\sqrt{7-2(x+1)^2}}$$

Let $u = x+1 \Rightarrow du = dx$

Let $\sqrt{2}u = \sqrt{7} \sin \theta \Rightarrow du = \frac{\sqrt{7}}{\sqrt{2}} \cos \theta d\theta$

$$= \int \frac{du}{\sqrt{7-2u^2}} = \sqrt{\frac{7}{2}} \int \frac{\cos \theta d\theta}{\sqrt{7-7\sin^2 \theta}} = \frac{\sqrt{7}}{\sqrt{2}} \int \frac{\cos \theta d\theta}{\sqrt{7} \sqrt{1-\sin^2 \theta}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{\cos \theta d\theta}{\sqrt{\cos^2 \theta}} = \frac{1}{\sqrt{2}} \int d\theta$$

$$= \frac{1}{\sqrt{2}} \theta + c$$

$$\therefore \frac{\sqrt{2}}{\sqrt{2}} u = \sin \theta \Rightarrow \sin^{-1} \left(\sqrt{\frac{2}{7}} u \right) = \theta.$$

$$= \frac{1}{\sqrt{2}} \sin^{-1} \left(\sqrt{\frac{2}{7}} u + c \right)$$

$$= \frac{1}{\sqrt{2}} \sin^{-1} \left(\sqrt{\frac{2}{7}} (x-3) \right) + c.$$

Integration by parts

$$\int u dv = uv - \int v du.$$

Ex: Evaluate: $\int x^2 \cdot e^{-x} dx$

Sol let $u = x^2 \Rightarrow du = 2x dx$

$$dv = e^{-x} dx \Rightarrow v = \int e^{-x} dx = -e^{-x}$$

So that $\int x^2 e^{-x} dx = \int u dv = uv - \int v du$

$$= -x^2 e^{-x} - 2 \int x e^{-x} dx$$

to find $\int x e^{-x} dx$

let $u = x \Rightarrow du = dx$

$$dv = e^{-x} dx \Rightarrow v = \int e^{-x} dx = -e^{-x}.$$

So that $\int x e^{-x} dx = \int u dv = uv - \int v du.$

$$= -x e^{-x} + \int e^{-x} dx$$

$$= -x e^{-x} - e^{-x} + c_1$$

$$\therefore \int x^2 e^{-x} dx = -x^2 e^{-x} + 2(-x e^{-x} - e^{-x} + c_1)$$

$$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + 2c_1$$

$$= -(x^2 + 2x + 2) e^{-x} + c \quad -63- \quad , \quad c_2 = 2c_1$$

Ex: Evaluate.

محاضرات في الرياضيات
كلية الهندسة / المرحلة الأولى
مهاجد أ. د. عبد الجليل

$$\int \ln x \, dx$$

Sol: let $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$$dv = dx \Rightarrow v = \int dx = x$$

$$\begin{aligned} \text{So that } \int \ln x \, dx &= \int u \, dv = uv - \int v \, du \\ &= x \ln x - \int x \frac{1}{x} dx \\ &= x \ln x - \int dx = x \ln x - x + c \end{aligned}$$

Ex: Evaluate: $\int e^x \cos x \, dx$.

Sol let $u = e^x \Rightarrow du = e^x dx$

$$dv = \cos x \, dx \Rightarrow v = \sin x.$$

$$\therefore \int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx$$

let $u = e^x \Rightarrow du = e^x dx$

$$dv = \sin x \, dx \Rightarrow v = -\cos x$$

$$\begin{aligned} \int e^x \cos x \, dx &= e^x \sin x - [-e^x \cos x + \int e^x \cos x \, dx] \\ &= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx \end{aligned}$$

$$2 \int e^x \cos x \, dx = e^x \sin x + e^x \cos x.$$

$$\therefore \int e^x \cos x \, dx = \frac{e^x \sin x + e^x \cos x}{2} + c$$

Ex: Evaluate $\int x^3 e^x \, dx$

Sol By using tabular method:

$$\int x^3 e^x \, dx = x^3 e^x - 3x^2 e^x + 6x e^x - 6 e^x + c$$

$+x^3$	e^x
$-3x^2$	e^x
$+6x$	e^x
-6	e^x
0	e^x

Ex: Evaluate

$$\int \frac{x e^x dx}{(x+1)^2}$$

محاضرات في التفاضل
كلية الطب / المرحلة الأولى
مهنا م. أ. د. نورا عبد الجليل

Sol let $u = x e^x \Rightarrow du = (x e^x + e^x) dx$
 $dv = (x+1)^{-2} dx \Rightarrow v = \frac{-1}{x+1}$

$$du = e^x (x+1) dx$$

$$\begin{aligned} \int \frac{x e^x dx}{(x+1)^2} &= \frac{-x e^x}{(x+1)} + \int \frac{e^x (x+1) dx}{x+1} \\ &= \frac{-x e^x}{(x+1)} + \int e^x dx \\ &= \frac{-x e^x}{(x+1)} + e^x + C \end{aligned}$$

Integration by partial Fraction

Ex: Evaluate

$$\int \frac{x^3 dx}{x^2 - 3x + 2}$$

Sol

$$= \int \left[x+3 + \frac{7x-6}{x^2-3x+2} \right] dx$$

$$= \frac{x^2}{2} + 3x + \int \frac{7x-6}{(x-1)(x-2)} dx$$

$$\frac{7x-6}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$$

$$\frac{7x-6}{(x-1)(x-2)} = \frac{A(x-2) + B(x-1)}{(x-1)(x-2)}$$

$$7x-6 = Ax - 2A + Bx - B$$

$$7x - 6 = (A+B)x - 2A - B$$

$$\therefore A+B = 7$$

$$-2A - B = -6$$

$$\therefore A = -1, \quad B = 8$$

$$\int \frac{(7x-6) dx}{(x-1)(x-2)} = \int \frac{-1 dx}{x-1} + \int \frac{8 dx}{x-2}$$

$$= -\ln|x-1| + 8\ln|x-2| + C.$$

$$\therefore \int \frac{x^3 dx}{x^2-3x+2} = \frac{x^2}{2} + 3x - \ln|x-1| + 8\ln|x-2| + C$$

Ex.: Evaluate $\int \frac{\cos \theta d\theta}{\sin^2 \theta + 4\sin \theta - 5}$

Sol Let $x = \sin \theta \Rightarrow dx = \cos \theta d\theta.$

$$= \int \frac{dx}{x^2-4x-5} = \int \frac{dx}{(x+1)(x-5)}$$

$$\frac{1}{(x+1)(x-5)} = \frac{A}{(x+1)} + \frac{B}{(x-5)}$$

$$\therefore A = -\frac{1}{6}, \quad B = \frac{1}{6}$$

$$\int \frac{dx}{(x+1)(x-5)} = -\frac{1}{6} \int \frac{dx}{x+1} + \frac{1}{6} \int \frac{dx}{x-5}$$

$$= -\frac{1}{6} \ln|x+1| + \frac{1}{6} \ln|x-5| + C$$

$$= -\frac{1}{6} \ln|\sin \theta + 1| + \frac{1}{6} \ln|\sin \theta - 5| + C$$

~~~~~

محاضرات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
صهيب احمد ا. د. زياد عبد الجليل

Def:

Any equation involve derivative is called Differential equation

معادلات في المتغيرات  
كثيرة المتغيرات / المعادلات التفاضلية  
مما ورد أ. د. محمد عبد الحليم

Ex: 1)  $\frac{dy}{dx} = \frac{x}{y}$

2)  $(x+y) dy = xy dx$

are D. Es.

Ex: Solve: D.E.  $\frac{dy}{dx} = y^2 - 5y + 6$

Sol  $\int \frac{dy}{y^2 - 5y + 6} = \int dx$

$= \int \frac{dy}{(y-2)(y-3)} = \int dx$

$\frac{1}{(y-2)(y-3)} = \frac{A}{(y-2)} + \frac{B}{(y-3)}$

$\therefore A = -1, B = 1$

$\int \frac{A dy}{(y-2)} + \int \frac{B dy}{(y-3)} = \int dx$

$= \int \frac{-1 dy}{(y-2)} + \int \frac{dy}{y-3} = \int dx$

$-\ln |y-2| + \ln |y-3| = x \Rightarrow \ln \left| \frac{y-3}{y-2} \right| = x$



# Integrals involving rational exponents

محاضرات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
م. د. أحمد عبد الجليل

Ex: Evaluate  $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$

Sol let  $u = x^{\frac{1}{6}} \Rightarrow u^6 = x \Rightarrow dx = 6u^5 du$ .

$$\begin{aligned} & \int \frac{6u^5 du}{u^3 + u^2} \\ &= 6 \int \frac{u^5 du}{u^2(u+1)} = 6 \int \frac{u^3 du}{u+1} \end{aligned}$$

$$\begin{array}{r} u^2 - u + 1 \\ \hline u+1 \overline{) \begin{array}{r} u^3 \\ +u^3 + u^2 \\ \hline -u^2 + u \\ +u + 1 \\ \hline -1 \end{array}} \end{array}$$

$$\begin{aligned} &= 6 \int \left[ u^2 - u + 1 - \frac{1}{u+1} \right] du \\ &= 6 \int \left[ \frac{u^2}{2} - \frac{u^2}{2} + u - \ln|u+1| \right] + C \\ &= 2 \frac{1}{x^2} - 3 \frac{1}{x^3} + 6 \frac{1}{x^{\frac{1}{6}}} - 6 \ln \left| \frac{1}{x^{\frac{1}{6}}} + 1 \right| + C. \end{aligned}$$

Ex  $\int \frac{dx}{x - x^{\frac{3}{5}}}$

let  $u = x^{\frac{1}{5}} \Rightarrow u^5 = x \Rightarrow 5u^4 du = dx$

$$\begin{aligned} & \int \frac{5u^4 du}{u^5 - u^3} = 5 \int \frac{u^4 du}{u^3(u^2 - 1)} = 5 \int \frac{u du}{u^2 - 1} \\ &= \frac{5}{2} \int \frac{2u du}{(u^2 - 1)} = \frac{5}{2} \ln|u^2 - 1| + C \\ &= \frac{5}{2} \ln \left| x^{\frac{2}{5}} - 1 \right| + C \end{aligned}$$



## Integrals involving rational expression in $\sin$ and $\cos$

عامرات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
مدرس أ. د. زيد عبد الجليل

Ex Evaluate

$$\int \frac{dx}{1 + \sin x + \cos x}$$

Sol Let  $x = 2 \tan^{-1} u \Rightarrow dx = \frac{2du}{1+u^2}$

$$\sin x = \frac{2u}{1+u^2}, \quad \cos x = \frac{1-u^2}{1+u^2}$$

$$\int \frac{\frac{2du}{1+u^2}}{1 + \frac{2u}{1+u^2} + \frac{1-u^2}{1+u^2}} = \int \frac{\frac{2du}{1+u^2}}{\frac{1+u^2+2u+1-u^2}{1+u^2}}$$

$$= \int \frac{2du}{2+2u} = \int \frac{du}{1+u}$$

$$= \ln |1+u| + C = \ln \left| 1 + \tan \frac{x}{2} \right| + C$$

## Integration power of sine and cosine

Ex: Evaluate:  $\int \sin^3 x \cos^2 x dx$

The power is odd and even.

$$= \int \sin^2 x \sin x \cos^2 x dx$$

$$= \int (1 - \cos^2 x) \sin x \cos^2 x dx$$

Let  $u = \cos x \Rightarrow du = -\sin x dx$

$$= - \int (1 - u^2) u^2 du = - \int (u^2 - u^4) du$$

$$= \frac{-u^3}{3} + \frac{u^5}{5} + C = \frac{-\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$$

$$\sin a \cos b = \frac{1}{2} [\sin(a-b) + \sin(a+b)]$$

$$= \frac{1}{2} [\cos(a-b) - \cos(a+b)]$$

$$= \frac{1}{2} [\cos(a-b) + \cos(a+b)]$$

مخاضات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
مادة أ. د. زياد عبد الحليم

Ex: Evaluate  $\int \sin 7x \cos 3x dx$

Sol  $\int \frac{1}{2} [\sin(4x) + \sin(10x)] dx$   
 $= \frac{1}{2} \int [\sin(4x)] dx + \frac{1}{2} \int [\sin(10x)] dx$   
 $= \frac{1}{8} \int [\sin(4x)] 4 \cdot dx + \frac{1}{20} \int \sin(10x) \cdot 10 dx$   
 $= -\frac{1}{8} \cos 4x - \frac{1}{20} \cos 10x + C.$

Ex: Evaluate  $\int \sin x \cos \frac{x}{2} dx$   
 $= \int \frac{1}{2} [\sin(\frac{x}{2}) + \sin(\frac{3x}{2})] dx$   
 $= \frac{1}{2} \int [\sin(\frac{x}{2})] dx + \frac{1}{2} \int [\sin(\frac{3x}{2})] dx$   
 $= -\cos \frac{x}{2} - \frac{1}{3} \cos \frac{3x}{2} + C.$

Integrating power of secant and tangent

Reduction formula.

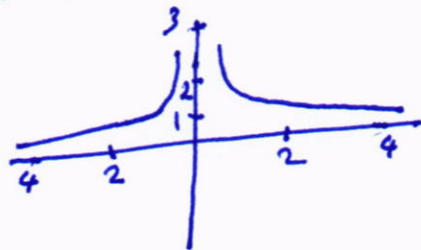
$$\int \sec^n x dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x dx$$

$$\int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx$$

# Improper Integrals

محاضرات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
م. ابراهيم أ. د. زياد عبد الجليل

1)  $\int_a^{\infty} f(x) dx$  or  $\int_{-\infty}^b f(x) dx$  are called improper integrals.



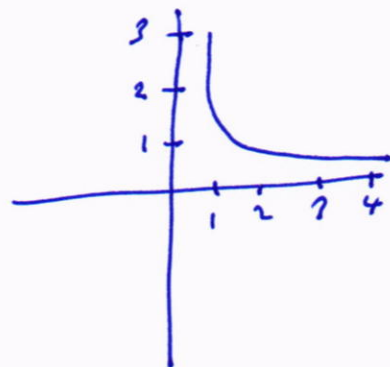
Ex: Find  $\int_1^{\infty} \frac{dx}{x^2}$

Sol  $\lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x^2} = \lim_{b \rightarrow \infty} \left[ -\frac{1}{x} \right]_1^b$

$= \lim_{b \rightarrow \infty} \left[ -\frac{1}{b} + 1 \right] = \lim_{b \rightarrow \infty} \left[ 1 - \frac{1}{b} \right] = 1$

and the integral is converges to 1.

Ex: Evaluate  $\int_1^{\infty} \frac{dx}{x}$



Sol  $\lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x}$

$= \lim_{b \rightarrow \infty} \left[ \ln x \right]_1^b$

$= \lim_{b \rightarrow \infty} [\ln b - \ln 1] = \infty$

The integral is diverges

Ex: Evaluate  $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

Sol  $\int_{-\infty}^0 \frac{dx}{1+x^2} + \int_0^{\infty} \frac{dx}{1+x^2}$

$= \lim_{b \rightarrow -\infty} \int_b^0 \frac{dx}{1+x^2} + \lim_{a \rightarrow \infty} \int_0^a \frac{dx}{1+x^2}$



محاذات في الرياضيات  
كلية الهندسة / المرحلة الثانية  
م. أ. د. محمد عبد الجليل

$$= \lim_{b \rightarrow -\infty} [\tan^{-1} x]_b^0 + \lim_{a \rightarrow \infty} [\tan^{-1} x]_0^a$$

$$= \lim_{b \rightarrow -\infty} [\tan^{-1} 0 + \tan^{-1} b] + \lim_{a \rightarrow \infty} [\tan^{-1} a - \tan^{-1} 0]$$

$$= -\left(\frac{\pi}{2}\right) + \frac{\pi}{2} = \pi$$

the integrals is converges to  $\pi$ .

Ex: Evaluate:  $\int_1^4 \frac{dx}{(x-2)^{2/3}}$ , this integral is improper because:  $f(x) \rightarrow \infty$  as  $x \rightarrow 2$  and  $2 \in [1, 4]$ .

Sol

$$\int_1^4 \frac{dx}{(x-2)^{2/3}} = \int_1^2 \frac{dx}{(x-2)^{2/3}} + \int_2^4 \frac{dx}{(x-2)^{2/3}}$$

$$= \lim_{a \rightarrow -2} \int_1^a (x-2)^{-2/3} dx + \lim_{b \rightarrow +2} \int_b^4 (x-2)^{-2/3} dx$$

$$= \lim_{a \rightarrow -2} \left[ 3(x-2)^{1/3} \right]_1^a + \lim_{b \rightarrow +2} \left[ 3(x-2)^{1/3} \right]_b^4$$

$$= \lim_{a \rightarrow -2} [3(a-2)^{1/3} - 3] + \lim_{b \rightarrow 2} [3(2)^{1/3} - 3(b-2)^{1/3}]$$

$$= 3\sqrt[3]{2}$$

the integral is converges.

## L'HOPITAL'S RULE

Theorem:  
if  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$  or  $\frac{\infty}{\infty}$   
then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

Ex: Find  $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$   $(\frac{0}{0})$

Sol by L'HÔPITAL Rule

محاور في الرياضيات  
لغة الهندسة / الهندسة الزاوية  
مناداد أ. د. زياد عبد الجليل

$$\lim_{x \rightarrow 0} \frac{2 \cos 2x}{1} = 2 \cdot 1 = 2$$

Ex: Find  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x^3}$   $(\frac{0}{0})$

by L'hospital Rule:

$$\lim_{x \rightarrow 0} \frac{e^x}{3x^2} = \frac{1}{0} = \infty$$

The  $e$  does not exist.

Ex:  $\lim_{x \rightarrow \infty} \frac{x^2 + x - 1}{2x^2 + 5}$   $(\frac{\infty}{\infty})$

Sol by L'hospital Rule.

$$= \lim_{x \rightarrow \infty} \frac{2x + 1}{4x} = \frac{\infty}{\infty}$$

$$= \lim_{x \rightarrow \infty} \frac{2}{4} = \frac{1}{2}$$

Ex: Find  $\lim_{x \rightarrow \infty} \frac{\ln(1 + \frac{3}{x})}{\sin(\frac{2}{x})}$   $(\frac{0}{0})$

Sol by hospital Rule.

$$\lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \frac{3}{x}} \cdot \frac{-3}{x^2}}{\cos(\frac{2}{x}) \cdot \frac{-2}{x^2}} = \frac{3}{2}$$

=====

The Form  $(0, \infty) \rightarrow \frac{0}{0}$  or  $\frac{\infty}{\infty}$

محاضرات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
م. اسد أ. د. رضا ديب المصيل

Ex: Find  $\lim_{x \rightarrow 0^+} x \ln x$   $(0, \infty)$ .

Sol

$$= \lim_{x \rightarrow 0} \frac{\ln x}{\frac{1}{x}} = \frac{\infty}{\infty}$$

by HOPITAL Rule.

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x \rightarrow 0} (-x) = 0.$$

The Form  $(+\infty, -\infty) \rightarrow \frac{0}{0}$  or  $\frac{\infty}{\infty}$

Ex: Find  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x)$   $(+\infty, -\infty)$

Sol

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x) \cdot \frac{\sqrt{x^2 + x} + x}{\sqrt{x^2 + x} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 + x - x^2}{\sqrt{x^2 + x} + x} = \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + x} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x}} + 1} = \frac{1}{2}$$

Ex: Find  $\lim_{x \rightarrow \infty} [x - \ln(x^2 + 1)]$   $(\infty, -\infty)$

Sol

$$\lim_{x \rightarrow \infty} [\ln e^x - \ln(x^2 + 1)]$$

$$= \lim_{x \rightarrow \infty} \left[ \ln \frac{e^x}{x^2 + 1} \right]$$

$$= \ln \left[ \lim_{x \rightarrow \infty} \frac{e^x}{x^2 + 1} \right] \frac{\infty}{\infty}$$

by HÔPITAL Rule

$$= \ln \left[ \lim_{x \rightarrow \infty} \frac{e^x}{2x} \right] \frac{\infty}{\infty}$$

by HÔPITAL Rule

$$= \ln \left[ \lim_{x \rightarrow \infty} \frac{e^x}{2} \right] = \ln \infty = \infty$$



# The Form $(0, \infty, 1)$

محاضرات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
م. م. م. أ. د. ن. ع. الجليل

Ex: Find  $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}}$

Sol: let  $y = (1+x)^{\frac{1}{x}}$

$$\ln y = \frac{1}{x} \ln(1+x)$$

$$\lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} \frac{1}{x} \ln(1+x) \quad (0, \infty)$$

$$= \lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{x} \quad \frac{0}{0} \text{ by H\o{P}ITAL Rule}$$

$$\ln \lim_{x \rightarrow 0^+} y = \lim_{x \rightarrow 0^+} \frac{\frac{1}{1+x}}{1} = 1$$

$$\ln \lim_{x \rightarrow 0^+} y = 1$$

take  $e$  for both sides

$$\lim_{x \rightarrow 0^+} y = e^1 \rightarrow \lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}} = e$$

Ex Find  $\lim_{x \rightarrow 0^+} (e^x + x)^{\frac{1}{x}}$

Sol let  $y = (e^x + x)^{\frac{1}{x}}$

$$\ln y = \frac{1}{x} \ln(e^x + x)$$

$$\lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} \frac{1}{x} \ln(e^x + x) \quad (0, \infty)$$

$$\lim_{x \rightarrow 0^+} \ln y = \lim_{x \rightarrow 0^+} \frac{\ln(e^x + x)}{x} \quad \left(\frac{0}{0}\right) \text{ by Hopital Rule}$$

$$\ln \lim_{x \rightarrow 0^+} y = \lim_{x \rightarrow 0^+} \frac{\frac{e^x + 1}{e^x + 1}}{1} = 2$$

$$\ln \lim_{x \rightarrow 0^+} y = 2 \Rightarrow \lim_{x \rightarrow 0^+} y = e^2$$

$$\therefore \lim_{x \rightarrow 0^+} (e^x + x)^{\frac{1}{x}} = e^2$$

# Matrices and Determinants

محاضرات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
م. عبد الله بن عبد الجليل

Def: Any arrangement of the elements in the following form

$$[A] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

is called a matrix of order  $m \times n$   
where  $m$  = number of rows.  
and  $n$  = number of columns.

Equal matrices:

$$\begin{bmatrix} a & b & c \\ e & f & g \\ h & i & j \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -2 \\ 4 & -5 & 6 \end{bmatrix} \leftarrow 3 \times 3 \text{ matrix}$$

iff  $a=2, b=1, c=3, e=1, f=0, g=-2$   
 $h=4, i=-5, j=6.$

Addition and Subtraction:

$$\text{Let } A = \begin{bmatrix} 1 & 4 \\ 2 & 9 \\ 0 & 8 \end{bmatrix} + B = \begin{bmatrix} 4 & 5 \\ 2 & 0 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ 4 & 9 \\ 3 & 10 \end{bmatrix}$$

Note  $[A]$  and  $[B]$  must be of the same order

Multiplication by constant:

$$\text{Def: } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, k \in \mathbb{Z} \Rightarrow k \cdot A = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

Let  $[A]$ ,  $[B]$  be two matrices such that  
number of rows of  $[B]$  = number of columns of  $[A]$

$$\text{Then } [A]_{m \times n} \cdot [B]_{n \times k} = [C]_{m \times k}$$

Ex: Find  $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

$2 \times 3 \qquad 3 \times 1$

خاصات في المصفوفات  
كلية الهندسة / الجامعة الأردنية  
صالح د. أ. د. م. عبد الله

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2+3+6 \\ 0+4+15 \end{bmatrix} = \begin{bmatrix} 11 \\ 19 \end{bmatrix}$$

### Types of Matrices:

- 1 Square Matrix, if,  $m=n$ , Ex =  $\begin{bmatrix} 2 & 3 & -1 \\ 0 & -2 & 4 \\ -1 & 0 & 3 \end{bmatrix}$
- 2 Row matrix, if,  $m=1$ , Ex:  $[A] = \begin{bmatrix} 3 & -1 & 2 & 7 \end{bmatrix}$
- 3 Column matrix, if,  $n=1$ , Ex =  $[A] = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 3 \end{bmatrix}$
- 4 Identity matrix, if sq. matrix with unity main diagonal

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5 Zero matrix  $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Note in general  $A \cdot B \neq B \cdot A$ .

6 Symmetric matrix  $[A] = \begin{bmatrix} 2 & 3 & -5 \\ 3 & 1 & 2 \\ -5 & 2 & -1 \end{bmatrix}$ , is a

Square matrix about main diagonal.

- 7 Skew Symmetric matrix: is a Square matrix, and the main diagonal are zero

$$Ex = \begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 2 \\ 5 & -2 & 0 \end{bmatrix}$$

- 8 Upper triangular matrix: is a Square matrix with elements zero below the main diagonal:

$$Ex: [A] = \begin{bmatrix} 1 & 4 & -6 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$



g. Lower triangular matrix: -  
 is a square matrix with elements zero above the main diagonal  
 محاذات في المثلث  
 كليه اقلية / المثلث الاقل

Ex:  $[A] = \begin{bmatrix} 3 & 0 & 0 \\ 7 & 2 & 0 \\ 1 & 4 & -1 \end{bmatrix}$

10. Transpose of a matrix:

Let  $[A] = \begin{bmatrix} 3 & 5 \\ 6 & 4 \\ -2 & 1 \end{bmatrix}_{3 \times 2} \Rightarrow A^T = \begin{bmatrix} 3 & 6 & -2 \\ 5 & 4 & 1 \end{bmatrix}_{2 \times 3}$

System of simultaneous linear equations in a matrix form:

$$2x_1 - 3x_2 + 8x_3 = 6$$

$$x_1 + 7x_2 = -4$$

$$5x_1 + x_2 - x_3 = 5$$

in matrix form:

$$\begin{bmatrix} 2 & -3 & 8 \\ 1 & 7 & 0 \\ 5 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 5 \end{bmatrix}$$

$$A \cdot X = B$$

11. Symmetric matrix:

Ex:  $\begin{bmatrix} 7 & 72 & -1 \\ 2 & 5 & 4 \\ -1 & 4 & 3 \end{bmatrix}$

Note that  $A = A^T$

12. Diagonal matrix: is a square matrix with all elements zero except on the leading diagonal

Ex:  $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

13. Unit matrix: is a square matrix in which the elements on the leading diagonal are one, The unit matrix is denoted by  $I$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



# Determinants

خاصات المصفوفات  
للمصفوفة المربعة / المربعة  
محدد  $\hat{A}$  هو عدد محدد

if  $A$  is a square matrix, then  
 $\det A$  or  $|A|$  is a number calculated from  $A$   
and found as follows.

if  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  then  $|A| = ad - bc$

Ex: Let  $A = \begin{bmatrix} 3 & 1 \\ 4 & -2 \end{bmatrix} \Rightarrow |A| = 3 \times -2 - 1 \times 4 = -10$

Minors: is a determinant of order  $n-1$  obtained from  
 $A$  after deleting the  $i$ th row and  $j$ th column.

Ex: Let  $[A] = \begin{bmatrix} 1 & 2 & -4 \\ 2 & -5 & 6 \\ 13 & 6 & 7 \end{bmatrix}$

The minor of  $a_{11} = \begin{bmatrix} -5 & 6 \\ 6 & 7 \end{bmatrix}$

To Find  $\det A$ , Find the cofactor of any  
row or any column that has maximum zero  
and make and sum, Note for  $n=3$ , only  
quick calculation of  $\det A$ .

Ex: Let  $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \\ 7 & -8 & 9 \end{bmatrix}$  then.

$$\begin{vmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \\ 7 & -8 & 9 \end{vmatrix} = 1(45 + 84 + 96) - 105 - (-72) = 240.$$

Properties of Determinants:

1. if two rows (or columns) are identical then

$|A| = 0$  Ex:  $\begin{vmatrix} 1 & 2 & 3 \\ -1 & 4 & 7 \\ -1 & 4 & 7 \end{vmatrix} = 0$



2. Interchanging any two rows or columns changes the sign of the determinant.

مبادلات في الصفات  
أو الأعمدة / التبدل  
تغير إشارة المحدد

$$E = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 1 & 6 \\ 2 & 5 & 7 \end{vmatrix} = - \begin{vmatrix} 3 & 1 & 6 \\ 2 & -1 & 3 \\ 2 & 5 & 7 \end{vmatrix}$$

3.  $|A^T| = |A|$

$$\text{Ex: } \begin{vmatrix} 2 & -4 & 18 \\ 1 & 0 & 6 \\ 5 & -1 & 3 \end{vmatrix} = 2 \begin{vmatrix} 1 & -2 & 9 \\ 1 & 0 & 6 \\ 5 & -1 & 3 \end{vmatrix} = 2 \times 3 \begin{vmatrix} 1 & -2 & 3 \\ 1 & 0 & 2 \\ 5 & -1 & 1 \end{vmatrix}$$

$$\therefore |AB| = |A| \cdot |B|$$

### Grammer's Rule method

Solve the following system of Linear eqs.  
using Grammer's Rule.

$$-2x + 3y - z = 1$$

$$x + 2y - z = 4$$

$$-2x - y + z = -3$$

Sol  $\begin{bmatrix} -2 & 3 & -1 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$

$$A \cdot X = B.$$

$$|A| = \begin{vmatrix} -2 & 3 & -1 \\ 1 & 2 & -1 \\ -2 & -1 & 1 \end{vmatrix} = -2 \neq 0.$$

$$x = \frac{|A_1|}{|A|} = \frac{\begin{vmatrix} 1 & 3 & -1 \\ 4 & 2 & -1 \\ -3 & -1 & 1 \end{vmatrix}}{-2} = \frac{-4}{-2} = 2$$

$$y = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} -2 & 1 & -1 \\ 1 & 4 & -1 \\ -2 & -3 & 1 \end{vmatrix}}{-2} = \frac{-6}{-2} = 3$$

$$Z = \frac{|A_3|}{|A|} = \frac{\begin{vmatrix} -2 & 3 & 1 \\ 1 & 2 & 6 \\ -2 & -1 & -3 \end{vmatrix}}{-2} = \frac{-8}{-2} = 4$$

حاضرات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
م. أحمد أ. د. محمد عبد الجليل

### Adj of a matrix

Let  $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$  find  $\text{adj } A$ .

Sol  $\text{adj } A = \begin{bmatrix} -18 & 6 & -10 \\ 17 & -10 & -1 \\ -6 & -2 & -28 \end{bmatrix}$

### Inverse of a Square Matrix

if  $A$  is  $n \times n$  matrix and  $|A| \neq 0$  then

$$A^{-1} = \frac{\text{adj } A}{|A|}, \text{ to find } A^{-1}$$

1 Find  $|A| \neq 0$ .

2 Find  $\text{cof } A$

3 Find  $\text{adj } A = [\text{cof } A]^T$

4 Find  $A^{-1} = \frac{\text{adj } A}{|A|}$

Ex: Find  $A^{-1}$  if  $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$

Sol 1.  $|A| = -94 \neq 0$ .

2.  $\text{cof } A = \begin{bmatrix} -18 & 17 & -6 \\ -6 & -10 & -2 \\ -10 & -1 & 28 \end{bmatrix}$

3  $\text{adj } A = [\text{cof } A]^T = \begin{bmatrix} -18 & -6 & -10 \\ 17 & -10 & -1 \\ -6 & -2 & 28 \end{bmatrix}$

$$4: A^{-1} = \frac{\text{adj}}{|A|} = \frac{1}{-94} \begin{bmatrix} -18 & -6 & -10 \\ 17 & -10 & -1 \\ -6 & -2 & 28 \end{bmatrix}$$

عناصر المصفوفة  
كلية الهندسة / المرحلة الأولى  
مادة الرياضيات / الجبر الخطي

To check  $A \cdot A^{-1} = A^{-1} \cdot A = I$ .

H.W.

Solution of set of L.E. by using inverse matrix method

Ex Solve the system of L.E using inverse Matrix Method.

$$\begin{aligned} x - 2y &= 1 \\ 2x + 3y + z &= 7 \\ -x + 2z &= 8 \end{aligned}$$

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & 1 \\ -1 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 8 \end{bmatrix}$$

1  $|A| = 16 \neq 0$  check.

2  $\text{cof } A$ .  $A_{11} = 6$ ,  $A_{12} = -5$ ,  $A_{13} = 3$   
 $A_{21} = 4$ ,  $A_{22} = 2$ ,  $A_{23} = -(-2) = 2$   
 $A_{31} = +(-2)$ ,  $A_{32} = (-1)$ ,  $A_{33} = (+)7$ .

$$\text{cof } A = \begin{bmatrix} 6 & -5 & 3 \\ 4 & 2 & 2 \\ -2 & -1 & 7 \end{bmatrix}$$

3  $\text{adj } A = [\text{cof } A]^T = \begin{bmatrix} 6 & 4 & -2 \\ -5 & 2 & -1 \\ 3 & 2 & 7 \end{bmatrix}$

4  $A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{16} \begin{bmatrix} 6 & 4 & -2 \\ -5 & 2 & -1 \\ 3 & 2 & 7 \end{bmatrix}$

$$X = [A^{-1}] \cdot [B]$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{16} \begin{bmatrix} 6 & 4 & -2 \\ -5 & 2 & -1 \\ 3 & 2 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ 8 \end{bmatrix}$$



$$\therefore x = \frac{9}{8}, y = \frac{1}{16}, z = \frac{73}{16}$$

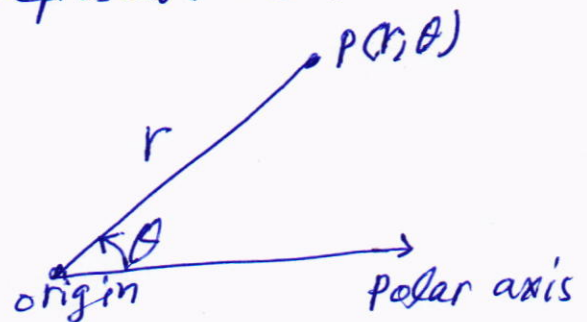
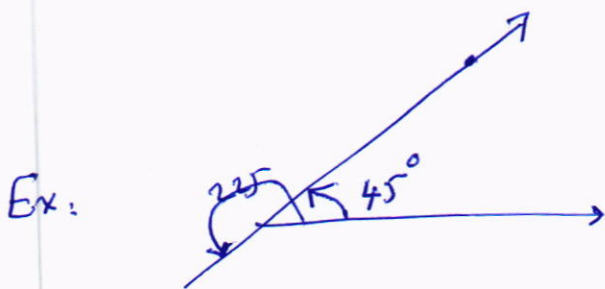
Note: to Find  $A^{-1}$  for

$$[A] = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

محاورات في الرياضيات  
كلية الطب / المرحلة الأولى  
مادة الفيزياء

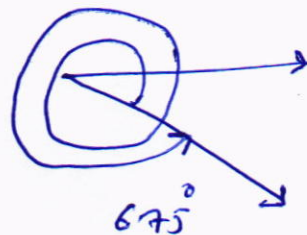
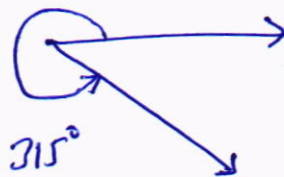
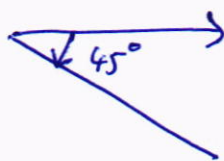
## Polar Coordinates.

The point  $p(r, \theta)$  can be represent. as:



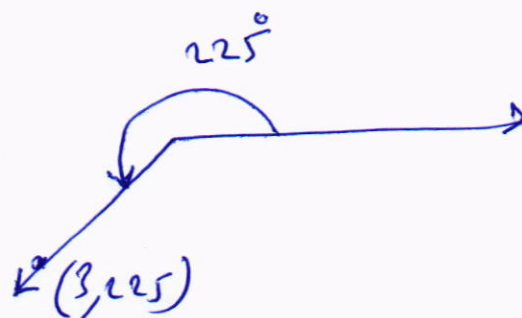
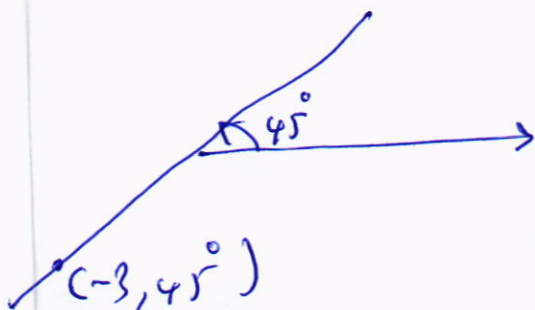
note 1:  $(r, \theta + n360^\circ) = (r, \theta - n360^\circ) = (r, \theta)$

Ex:  $(1, 45^\circ) = (1, 315^\circ) = (1, 675^\circ)$



note 2:  $(-r, \theta) = (r, \theta + 180^\circ)$

Ex:  $(3, 45^\circ) = (3, 225^\circ)$



محاضرات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
ص. ١٠٠ د. محمد عبد الحليم

using.

$$x = r \cos \theta.$$

$$\theta = \tan^{-1} \frac{y}{x}$$

Ex:

Sol

$$y = r \sin$$

∴ the

Ex:

$$r = \sqrt{x^2}$$

$$\theta = \tan^{-1}$$

Since  $C$

the poi

H-w :

1)

1)

2)  $F$

1)

Ex: c

Sol





$$y = b$$

$$r \sin \theta = b$$

$$r = \frac{b}{\sin \theta}$$

$$r = b \csc \theta$$

The general eq of the line is

$$Ax + By + c = 0$$

$$Ar \cos \theta + Br \sin \theta = -c$$

$$r(A \cos \theta + B \sin \theta) = -c$$

$$\therefore r = \frac{-c}{A \cos \theta + B \sin \theta}$$

Circles in polar co

$$r^2 + r_0^2 - 2rr_0 \cos(\theta - \theta_0) = a^2$$

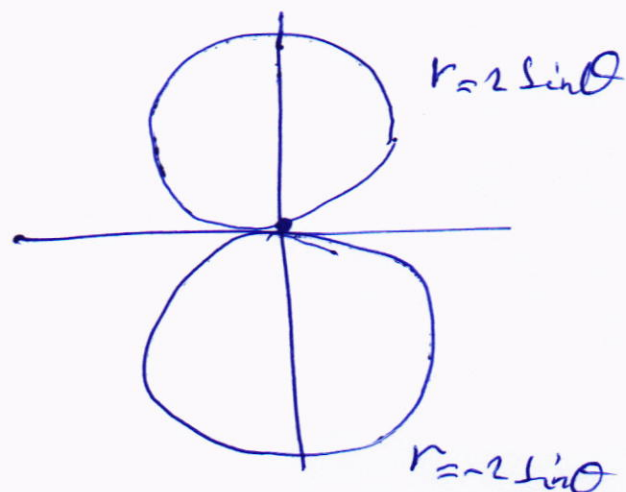
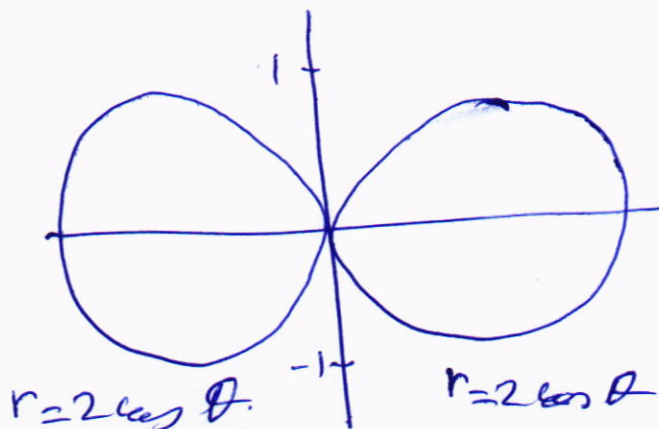
represent a circle of center

$(r, \theta_0)$  and  $r = a$ .

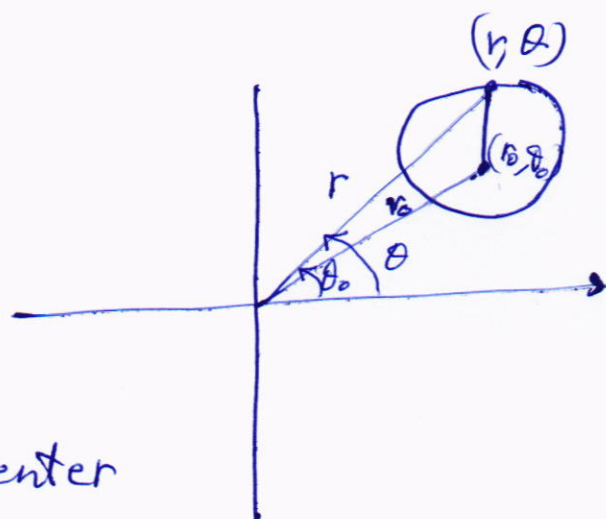
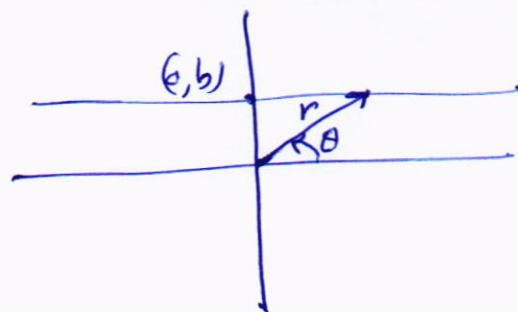
if the center  $(a, 0)$  or  $r_0 = a$ ,  $\theta_0 = 0$

$$r^2 - 2ar \cos \theta = 0$$

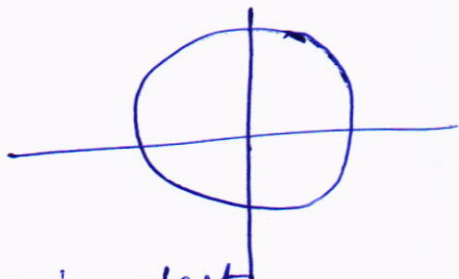
$$r = 2a \cos \theta$$



محاورات في الرافعات  
كلية الهندسة / المرحلة الأولى  
مهنايد أ. د. مراد عبد الجليل



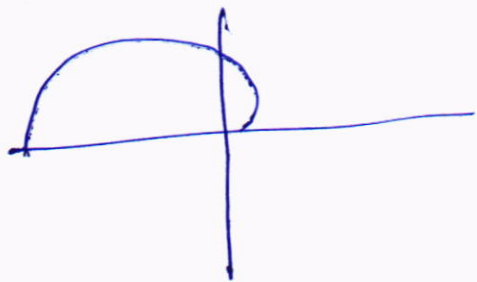
if the center  $(0,0)$  or  $r_0=0, \theta_0=0$  فاصلات في الدائريات  
 $r=a, a=2$  كلية الهندسة / الجامعة العراقية  
مادة أ. د. رياض الجليل



### Symmetric test:

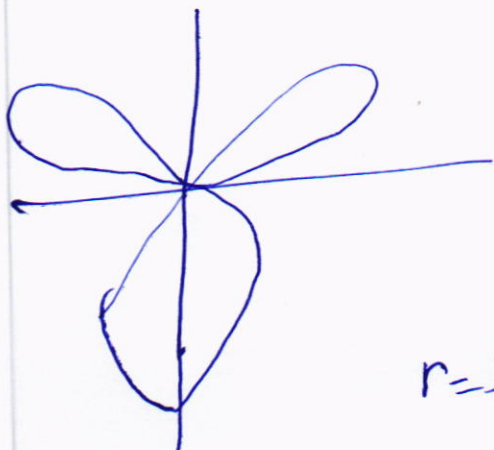
- 1) The curve  $r = f(\theta)$  symmetric about y-axis, if  $\theta$  replaced by  $-\theta$  and the eq equivalent.
- 2) The curve  $r = f(\theta)$  symmetric about x-axis if  $\theta$  replaced by  $\pi - \theta$  and the eq equivalent.
- 3) The curve  $r = f(\theta)$  is symmetric about the origin if  $r$  replaced by  $-r$ , and the eq - equivalent.

Ex: sketch the curve  $r = a(1 - \cos \theta)$  cardioids  
 it is symm about x-axis, so we draw from  $\theta = 0$  to  $\theta = \pi$

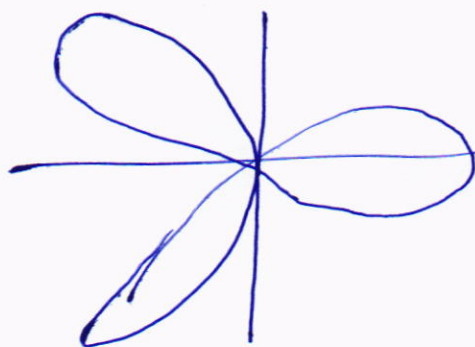


Ex: sketch the curve  
 $r = 5 \sin 3\theta$ .

Sol if  $n=3$  odd.  
 number of petals = 3



$$r = 5 \cos 3\theta$$



## Area in polar coo

محاور في الرياضيات  
على الخط / الرقعة السوداء  
منطقة التي هي في الداخل

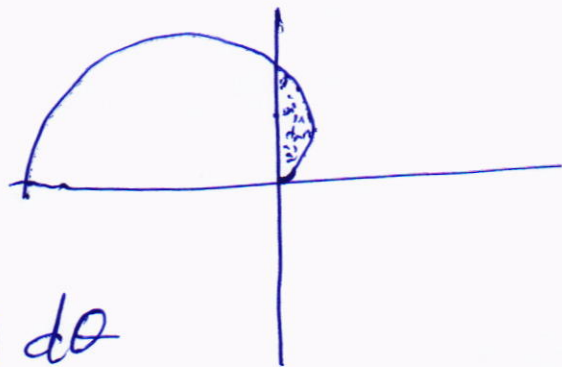
$$A = \int_{\alpha}^{\beta} \frac{1}{2} [f(\theta)]^2 d\theta = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta.$$

Ex: Find the area of the region in the first quadrant within the cardioid.

$$r = 1 - \cos \theta.$$

Sol

$$A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta = \frac{1}{2} \int_{\alpha}^{\beta} (1 - \cos \theta)^2 d\theta$$



$$= \frac{1}{2} \int_0^{\pi/2} (1 - 2\cos \theta + \cos^2 \theta) d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} (1 - 2\cos \theta + \frac{1}{2}(1 + \cos 2\theta)) d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} (\frac{3}{2} - 2\cos \theta + \frac{1}{2}\cos 2\theta) d\theta$$

$$= \frac{1}{2} \left[ \frac{3}{2}\theta - 2\sin \theta + \frac{1}{4}\sin 2\theta \right]_0^{\pi/2}$$

$$= \frac{3}{8}\pi - 1$$

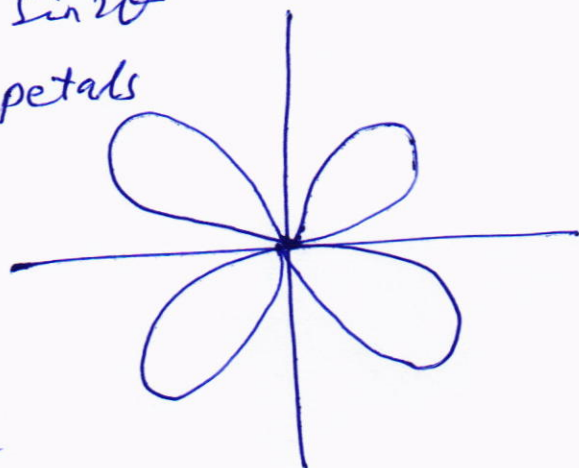
Ex: Find the area of the region enclosed by the rose curve  $r = \sin 2\theta$

Sol From symmetry of the petals

$$A = 4 \int_0^{\pi/2} \frac{1}{2} r^2 d\theta.$$

$$= 2 \int_0^{\pi/2} [\sin 2\theta]^2 d\theta$$

$$= 2 \int_0^{\pi/2} \frac{1}{2} (1 - \cos 4\theta) d\theta$$





$$= \left[ \theta - \frac{1}{4} \sin 4\theta \right]_0^{\pi/2} = \frac{\pi}{2}$$

محاور في الراسيات  
 على اظنه / المثلثات  
 ما شاء الله

Area between two polar curves

if  $r_1 = f_1(\theta)$  ,  $r_2 = f_2(\theta)$  then.

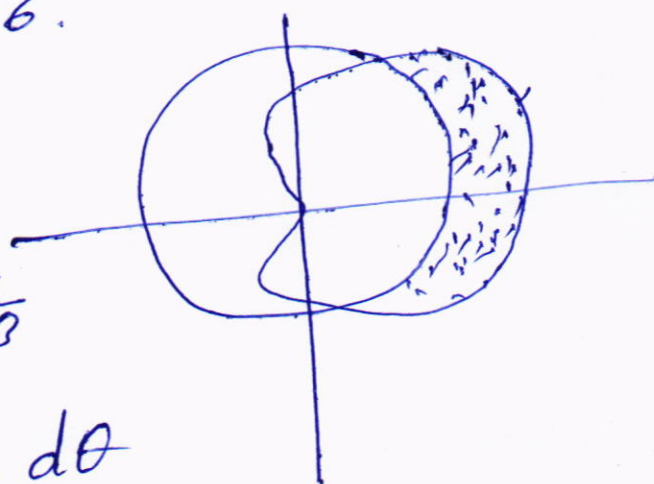
$$A = \frac{1}{2} \int_{\alpha}^{\beta} [r_1^2 - r_2^2] d\theta$$

Ex: Find the area of the region that is inside the cardioid  $r = 4 + 4\cos\theta$  and outside  $r = 6$ .

Sol  $6 = 4 + 4\cos\theta$

$2 = 4\cos\theta$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = -\frac{\pi}{3}, \theta = \frac{\pi}{3}$



$$A = \frac{1}{2} \int_{-\pi/3}^{\pi/3} [r_1^2 - r_2^2] d\theta$$

$$= \frac{1}{2} \int_{-\pi/3}^{\pi/3} [4 + 4\cos\theta]^2 - 6^2 d\theta$$

$$= \frac{1}{2} \int_{-\pi/3}^{\pi/3} [16 + 32\cos\theta + 16\cos^2\theta - 36] d\theta$$

$$= \int_{-\pi/3}^{\pi/3} [16\cos\theta + 8\cos^2\theta - 10] d\theta$$

$$= \int_{-\pi/3}^{\pi/3} \left[ 16\cos\theta + 8 \left[ \frac{1}{2} (1 + \cos 2\theta) \right] - 10 \right] d\theta$$

$$= \left[ 16\sin\theta + 4\theta + 2\sin 2\theta - 10\theta \right]_{-\pi/3}^{\pi/3} = 18\sqrt{3} - 4\pi$$

to Find the points of the intersection  
 حافظات في الرياضيات  
 كلية الهندسة / المرحلة الأولى  
 م. محمد أ. د. محمد الجليلي

Ex:  $r = 1 - \cos \theta$   
 $r = 1 + \cos \theta$

Sol  $1 - \cos \theta = 1 + \cos \theta$

$$2 \cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \theta = \frac{3\pi}{2}$$

and  $r = 1$

$\therefore$  the points of intersection are  $(1, \frac{\pi}{2}), (1, \frac{3\pi}{2})$ .

### Arc Length of polar curves

if  $r = f(\theta)$  is a polar curve such that  
 $f'(\theta)$  is continuous for

$$\alpha \leq \theta \leq \beta$$

then the arc length defined by.

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

Ex: Find the arc length of the polar curve  
 $r = e^{\theta}$  from  $\theta = 0$ ,  $\theta = 1$

Sol  $\because r = e^{\theta} \Rightarrow \frac{dr}{d\theta} = e^{\theta}$

$$\therefore L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

$$L = \int_0^1 \sqrt{e^{2\theta} + e^{2\theta}} d\theta = \int_0^1 \sqrt{2e^{2\theta}} d\theta$$

$$= \sqrt{2} \int_0^1 e^{\theta} d\theta = \sqrt{2} [e^{\theta}]_0^1 = \sqrt{2} (e - 1)$$

محاضرات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
م. أحمد م. دويهم

محاضرات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
م. أحمد م. دويهم

1) addition and subtraction.

Let  $z_1 = x_1 + iy_1$

$$z_2 = x_2 + iy_2$$

then  $z_1 + z_2 = (x_1 \pm x_2) + i(y_1 \pm y_2)$ .

2) Multiplication two complex numbers.

$$z_1 \cdot z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1).$$

Ex: let  $z_1 = 3 + 2i$

$$z_2 = -4 + 5i$$

then  $z_1 + z_2 = [3 + (-4)] + i[(2) + 5] = -1 + 7i$

$$z_1 - z_2 = [3 - (-4)] + i[2 - 5] = 7 - 3i$$

$$2. z_1 = 2(3+2i) = 6+4i$$

$$z_1 \cdot z_2 = (3+2i) + i(-4+5i)$$

$$= [(3x-4) - 2xj] + i[3xj + 2x-4]$$

$$= [-12 - 10] + i[15 - 8] = -22 + 7i$$

The conjugate of a complex number  $z = x + iy$  is the

Complex number  $\bar{z} = x - iy$

Properties: 1)  $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

$$2) \frac{\overline{\left(\frac{z_1}{z_2}\right)}}{\overline{z_2}} = \frac{\overline{z_1}}{\overline{z_2}}$$

3)  $2 \cdot \vec{z} = x^2 + y^2$

4)  $z + \bar{z} = 2x$

5)  $z - \bar{z} = 2iy$



Ex. Find:  $\frac{1+5i}{3+2i}$

مأمورات في الرياضيات  
كلية الهندسة / المرحلة الأولى  
م. اسرار أ. د. / مساعد أ. د.

Sol  $\frac{(1+5i)}{(3+2i)} \cdot \frac{(3-2i)}{(3-2i)}$

$$= \frac{[1 \times 3 - 5 \times -2 + i(1 \times -2) + 5 \times 3]}{9+4}$$

$$= \frac{13+13i}{13} = \frac{13(1+i)}{13} = 1+i$$

The magnitude and argument of Z

Each complex number Z can be written in polar form as  $Z = (r, \theta)$ , where.

$$r = |Z| = \sqrt{x^2 + y^2} \quad \theta = \arg Z = \tan^{-1} \frac{y}{x}$$

Ex. The complex number  $Z = \frac{\sqrt{3}}{2} + i\frac{1}{2}$  can be written in polar form.

$$r = |Z| = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 1$$

$$\theta = \tan^{-1} \frac{y}{x} = \tan^{-1} \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \tan^{-1} \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$$

$$\therefore (r, \theta) = \left(1, \frac{\pi}{6}\right)$$

$$Z = x + iy = r \cos \theta + ir \sin \theta = r(\cos \theta + i \sin \theta)$$

$$Z = r e^{i\theta}$$

where  $\boxed{e^{i\theta} = \cos \theta + i \sin \theta}$  Euler formula

if  $z_1 = r_1 e^{i\theta_1}$ ,  $z_2 = r_2 e^{i\theta_2}$  then

$$Z_1 \cdot Z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

Roots of complex numbers.

$$(Z)^{p/q} = r^{p/q} (e^{i(\frac{p}{q}(\theta + 2k\pi))}) \quad k=0, 1, 2, \dots, q-1$$

$$= r^{p/q} (\cos \frac{p}{q}(\theta + 2k\pi) + i \sin \frac{p}{q}(\theta + 2k\pi)).$$

Ex. Find the roots of  $\sqrt[4]{-8i} = (-8i)^{1/4}$

$$Z = x + iy = 0 - 8i, \quad x=0, \quad y=-8$$

$$r = |Z| = \sqrt{x^2 + y^2} = \sqrt{0 + (-8)^2} = 8$$

$$\theta = \tan^{-1} \frac{y}{x} = \tan^{-1} \frac{-8}{0} = -\frac{\pi}{2} = \frac{3\pi}{2}$$

$$Z^{p/q} = r^{p/q} (\cos \frac{p}{q}(\theta + 2k\pi) + i \sin \frac{p}{q}(\theta + 2k\pi)) \quad k=0, 1, 2, 3$$

$$Z_0 = 8^{1/4} (\cos \frac{1}{4}(\frac{3\pi}{2}) + i \sin \frac{1}{4}(\frac{3\pi}{2})) \quad k=0$$

$$Z_1 = 8^{1/4} (\cos \frac{1}{4}(\frac{3\pi}{2} + 2\pi) + i \sin \frac{1}{4}(\frac{3\pi}{2} + 2\pi)) \quad k=1$$

$$Z_2 = 8^{1/4} (\cos \frac{1}{4}(\frac{3\pi}{2} + 4\pi) + i \sin \frac{1}{4}(\frac{3\pi}{2} + 4\pi)) \quad k=2$$

$$Z_3 = 8^{1/4} (\cos \frac{1}{4}(\frac{3\pi}{2} + 6\pi) + i \sin \frac{1}{4}(\frac{3\pi}{2} + 6\pi)) \quad k=3$$

$$Z_3 = 8^{1/4} (\cos \frac{1}{4}(\frac{3\pi}{2} + 6\pi) + i \sin \frac{1}{4}(\frac{3\pi}{2} + 6\pi))$$

H.W Find the Root of  $(-1-i)^{4/5}$

### Complex Functions

$$1) \quad e^z = e^{x+iy} = e^x \cdot e^{iy} = e^x (\cos y + i \sin y) \\ = e^x (\cos y + i e^x \sin y)$$

$$2) \quad \ln z = \ln(re^{i\theta}) = \ln r + \ln e^{i\theta} \\ = \ln r + i\theta = \ln \sqrt{x^2 + y^2} + i \tan^{-1} \frac{y}{x}$$



محاضرات في الزايفيات

كلية الهندسة / المرحلة الأولى

مستاد أ. د. مزاحم الجليل

$$\begin{aligned} 3) \quad \sin z &= \sin(x+iy) \\ &= \sin x \cos iy + \sin iy \cos x \\ &= \sin x \cosh y + i \sin hy \cos x \end{aligned}$$



H.W: 1) Determine whether  $f$  and  $g$  are inverse Functions.

a)  $f(x) = 4x$ ,  $g(x) = \frac{1}{4}x$

Ans: Yes.

b)  $f(x) = x^4$ ,  $g(x) = \sqrt[4]{x}$

Ans No.

2) Solve for  $y$  if  $\ln(y-1) - \ln y = 2x$

Ans:  $y = \frac{1}{1-e^{2x}}$

3) Find  $\frac{dy}{dx}$  of  $x^2 + \sin 2x$

Ans  $e^{x^2 + \sin 2x} (2x + 2 \cos 2x)$

4) if  $\sinh x = \frac{-3}{4}$ , Find the value of the:

Ans  $\frac{-3}{4}$

5) Find  $y'$ , if  $y = \sinh^{-1} \sqrt{x}$

Ans:  $\frac{1}{2\sqrt{x(1+x)}}$

6) Evaluate  $\int 6 \cosh\left(\frac{x}{2} - \ln 3\right) dx$

Ans:  $12 \sinh\left(\frac{x}{2} - \ln 3\right)$

7) Evaluate the integrals in terms of:

a) inverse hyperbolic Function

b) natural logarithms.

of:  $\int_0^{2\sqrt{3}} \frac{dx}{\sqrt{4+x^2}}$

Ans a)  $\sinh^{-1}(\sqrt{3})$  b)  $\ln(\sqrt{3}+2)$

8) Evaluate  $\int \ln x dx$  Ans  $x \ln x - \int dx = x \ln x - x + c$

9) Evaluate  $\int \frac{dx}{2+2\sqrt{x}}$  Ans:  $\sqrt{x} - \ln|1+\sqrt{x}| + c$

10) Find  $y'$  if  $y = \ln \frac{(x^2+1)(x+3)^{\frac{1}{2}}}{x-1}$ , Ans:  $\frac{(x^2+1)(x+3)^{\frac{1}{2}}}{x-1} \left( \frac{2x}{x^2+1} + \frac{1}{2x+6} - \frac{1}{x-1} \right)$

11) Use L'Hôpital's Rule  $\lim_{x \rightarrow 0} \frac{3x - \sin x}{x}$  Ans: 2

12) Solve the systems of equations by a matrix method

$$x_1 + 2x_2 + x_3 = 4$$

$$3x_1 - 4x_2 - 2x_3 = 2$$

$$5x_1 + 3x_2 + 5x_3 = -1$$

Ans  $x_1 = 2$   
 $x_2 = 3$   
 $x_3 = -4$

(97)

محاضرات في الرياضيات  
 من إعداد أ. د. خالد عبد الحليم  
 كلية الطب - المرحلة الأولى