R.C.L SERIES CURRENT II

Thirteen lecture

Electrical engineering

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Department:

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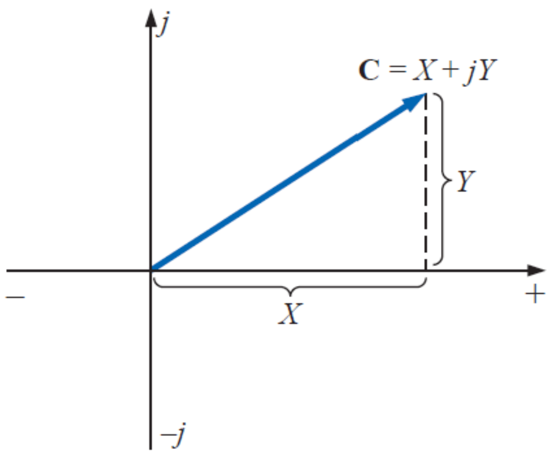
13.1 Complex Numbers

A complex number represents a point in a two-dimensional plane located with reference to two distinct axes

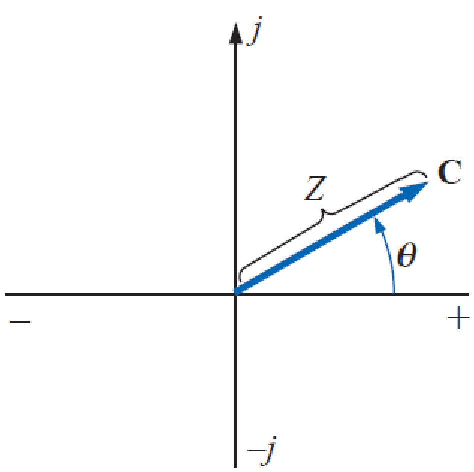
- The horizontal axis is called the real axis, while the vertical axis is called the imaginary axis.
- Two forms are used to represent a complex number: rectangular and polar.

1) **Rectangular Form**: The format for the rectangular form is



$$\boxed{\mathbf{C} = X + jY}$$

2) **Polar Form:** The format for the **polar form** is:



$$\boxed{\mathbf{C} = Z \angle \theta}$$

$$\boxed{-\mathbf{C} = -Z \angle \theta = Z \angle \theta \pm 180^\circ}$$

Mathematical Operations with Complex Numbers:

Complex numbers lend themselves readily to the basic mathematical operations of addition, subtraction, multiplication, and division.

$$\boxed{j = \sqrt{-1}}$$

$$\boxed{j^2 = -1} \qquad \boxed{\frac{1}{j} = -j}$$

1) Addition

$$\mathbf{C}_1 = \pm X_1 \pm j\,Y_1 \quad \text{and} \quad \mathbf{C}_2 = \pm X_2 \pm j\,Y_2$$

$$\boxed{\mathbf{C}_1 + \mathbf{C}_2 = (\pm X_1 \pm X_2) + j\,(\pm Y_1 \pm Y_2)}$$

EXAMPLE:

a. Add $\mathbf{C}_1 = 2 + j4$ and $\mathbf{C}_2 = 3 + j1$.
b. Add $\mathbf{C}_1 = 3 + j6$ and $\mathbf{C}_2 = -6 + j3$.

Solutions:

a)

$$\mathbf{C}_1 + \mathbf{C}_2 = (2 + 3) + j(4 + 1) = \mathbf{5 + j5}$$

b)

$$\mathbf{C}_1 + \mathbf{C}_2 = (3 - 6) + j(6 + 3) = \mathbf{-3 + j9}$$

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2) Multiplication

$$\mathbf{C}_1 = X_1 + j\,Y_1 \quad \text{and} \quad \mathbf{C}_2 = X_2 + j\,Y_2$$

$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = (X_1X_2 - Y_1Y_2) + j\,(Y_1X_2 + X_1Y_2)}$$

$$\mathbf{C}_1 = Z_1 \angle \theta_1 \quad \text{and} \quad \mathbf{C}_2 = Z_2 \angle \theta_2$$

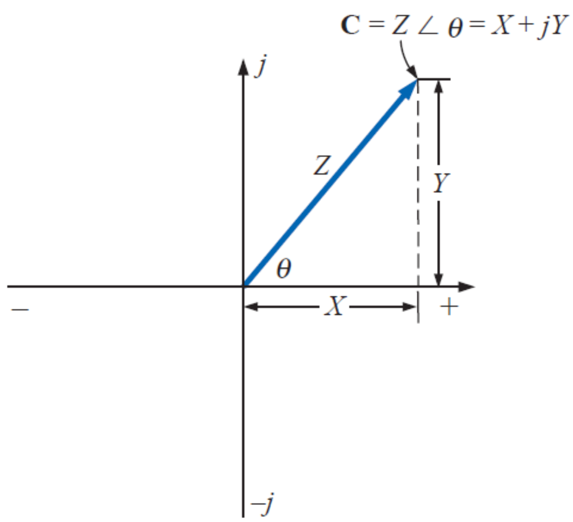
$$\boxed{\mathbf{C}_1 \cdot \mathbf{C}_2 = Z_1Z_2 \angle \theta_1 + \theta_2}$$

3) Division

$$\mathbf{C}_1 = Z_1 \angle\theta_1 \quad \text{and} \quad \mathbf{C}_2 = Z_2 \angle\theta_2$$

$$\boxed{\frac{\mathbf{C}_1}{\mathbf{C}_2} = \frac{Z_1}{Z_2} \underline{/\theta_1 - \theta_2}}$$

13.2 Conversion Between Forms



1) Rectangular to Polar

$$\boxed{Z = \sqrt{X^2 + Y^2}}$$

$$\boxed{\theta = \tan^{-1}\frac{Y}{X}}$$

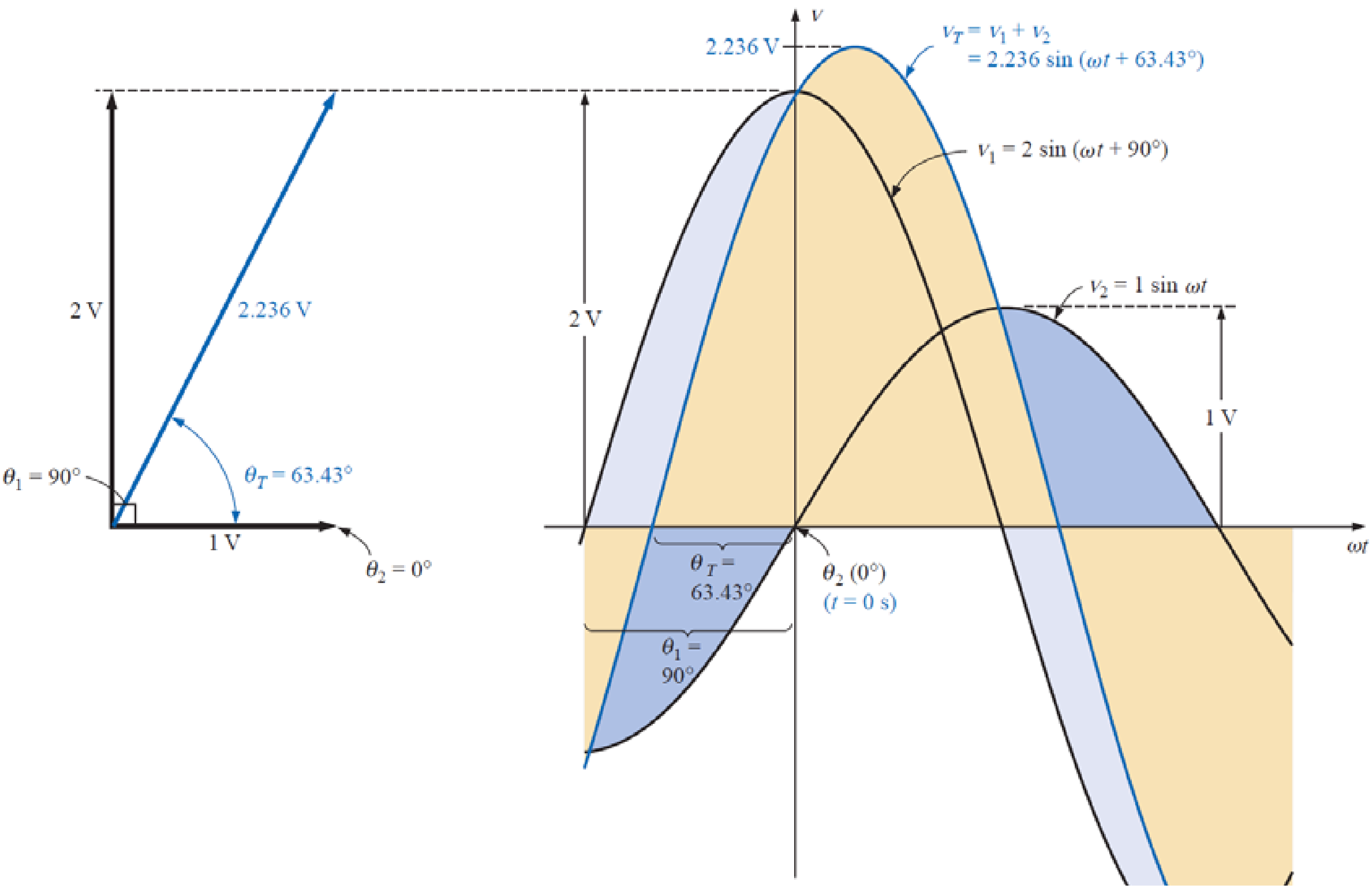
2) Polar to Rectangular

$$\boxed{X = Z\cos\theta}$$

$$\boxed{Y = Z\sin\theta}$$

13.3 Phasors :

The radius vector, having a constant magnitude (length) with one end fixed at the origin, is called a **phasor** (المتجه) when applied to electric circuits.



$$\boxed{e = Vm\sin(\omega t + \theta) \quad \Rightarrow \quad e = \frac{Vm}{\sqrt{2}}\angle\theta = 0.707\,Vm\angle\theta}$$

EXAMPLE: Convert the following from the time to the phasor domain:

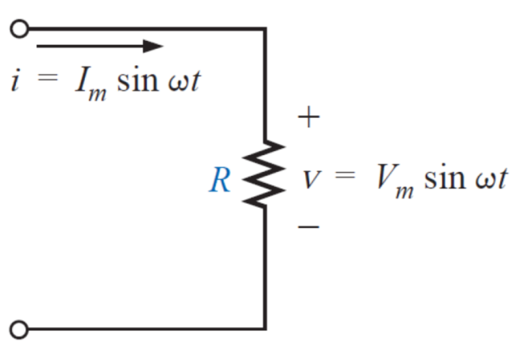
Time Domain	Phasor Domain
a. $\sqrt{2}(50)\sin\omega t$	$\mathbf{50\angle 0^\circ}$
b. $69.6\sin(\omega t + 72^\circ)$	$(0.707)(69.6)\angle 72^\circ = \mathbf{49.21\angle 72^\circ}$
c. $45\cos\omega t$	$(0.707)(45)\angle 90^\circ = \mathbf{31.82\angle 90^\circ}$

EXAMPLE: Write the sinusoidal expression for the following phasors if the frequency is 60 Hz:

Phasor Domain	Time Domain
a. $\mathbf{I} = 10\angle 30^\circ$	$i = \sqrt{2}(10)\sin(2\pi 60t + 30^\circ)$ and $i = \mathbf{14.14\sin(377t + 30^\circ)}$
b. $\mathbf{V} = 115\angle -70^\circ$	$v = \sqrt{2}(115)\sin(377t - 70^\circ)$ and $v = \mathbf{162.6\sin(377t - 70^\circ)}$

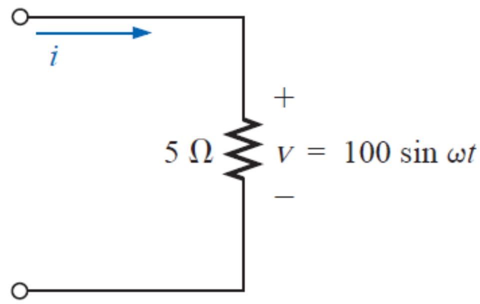
13.4 Impedance and The Phasor Diagram

1) Resistive Elements



$$\boxed{\mathbf{Z}_R = R\angle 0^\circ}$$

EXAMPLE: find the current I for the circuit. Sketch the waveforms of v and I.

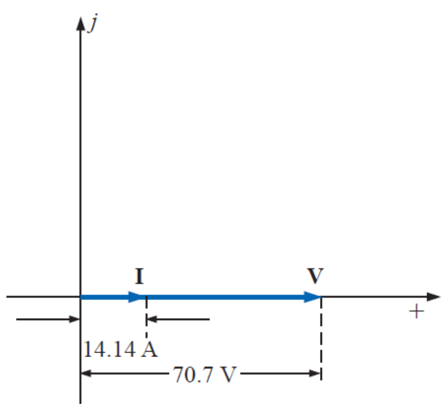


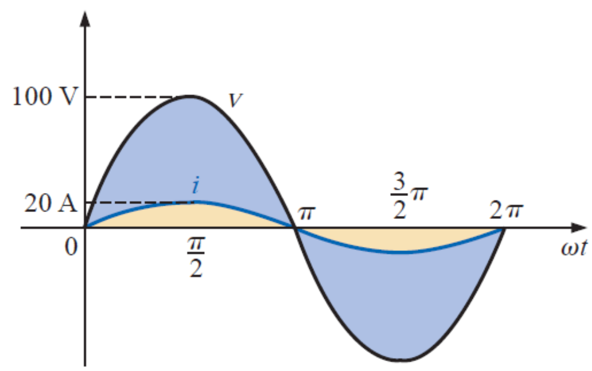
Solutions:

$v = 100 \sin \omega t \Rightarrow$ phasor form $\mathbf{V} = 70.71\text{ V} \angle 0°$

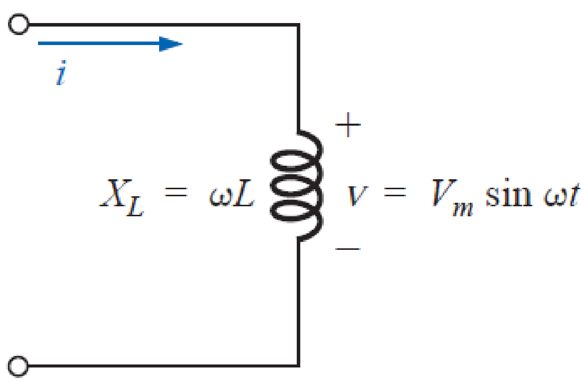
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_R} = \frac{V\angle\theta}{R\angle 0°} = \frac{70.71\text{ V}\angle 0°}{5\ \Omega\angle 0°} = 14.14\text{ A}\angle 0°$$

$$i = \sqrt{2}(14.14)\sin\omega t = \mathbf{20\ sin\ \omega t}$$



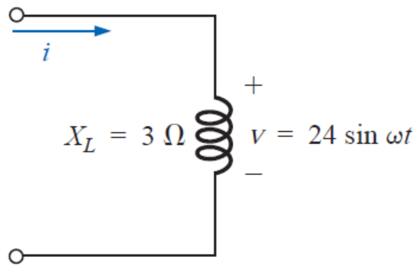


2) Inductive Reactance



$$\boxed{\mathbf{Z}_L = X_L \angle 90°}$$

EXAMPLE: find the current I for the circuit. Sketch the v and I curve

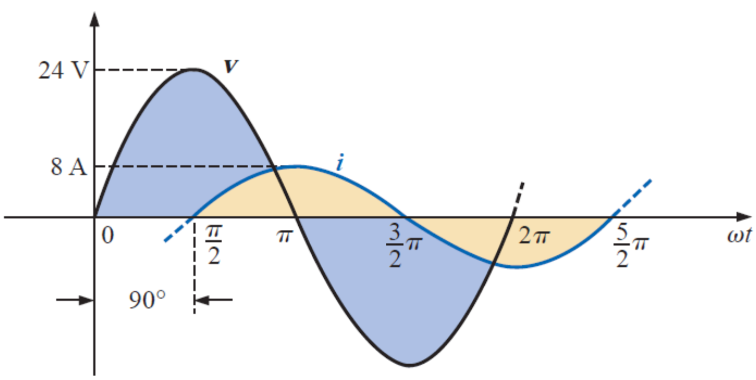


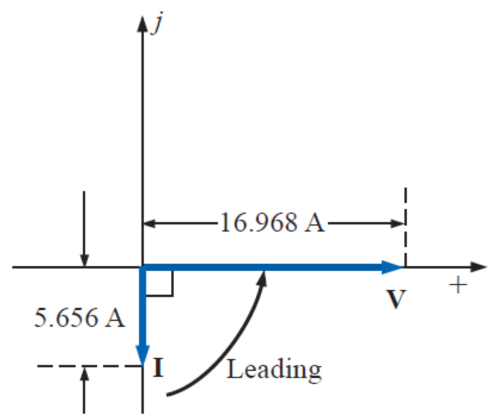
Solutions:

$v = 24 \sin \omega t \Rightarrow$ phasor form $\mathbf{V} = 16.968 \text{ V} \angle 0°$

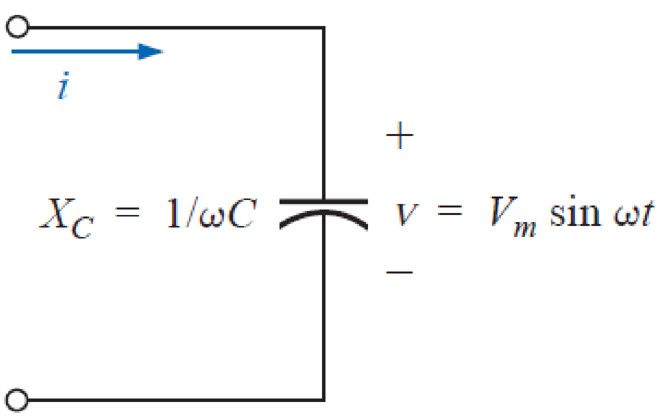
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_L} = \frac{V \angle \theta}{X_L \angle 90°} = \frac{16.968 \text{ V} \angle 0°}{3 \ \Omega \angle 90°} = 5.656 \text{ A} \angle -90°$$

$$i = \sqrt{2}(5.656) \sin(\omega t - 90°) = \mathbf{8.0 \sin(\omega t - 90°)}$$



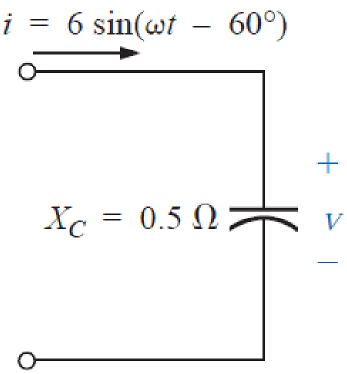


3) Capacitive Reactance



$$\boxed{\mathbf{Z}_C = X_C \angle -90°}$$

EXAMPLE: find the voltage v for the circuit. Sketch the v and I curves.

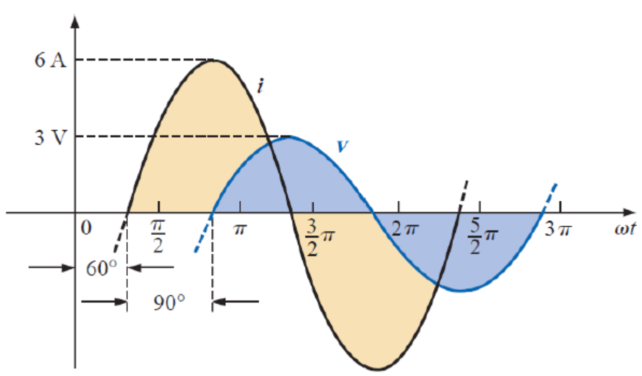


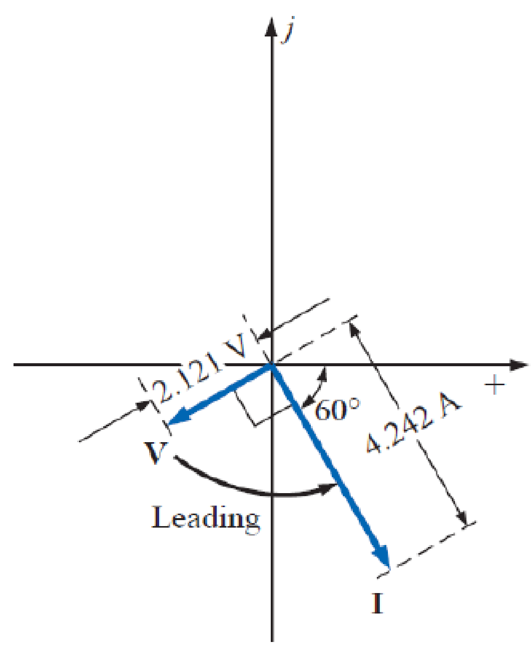
Solutions:

$i = 6 \sin(\omega t - 60°) \Rightarrow$ phasor notation $\mathbf{I} = 4.242 \text{ A} \angle -60°$

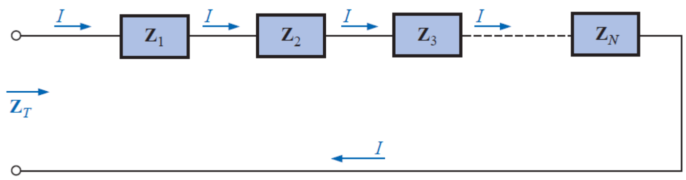
$\mathbf{V} = \mathbf{IZ}_C = (I \angle \theta)(X_C \angle -90°) = (4.242 \text{ A} \angle -60°)(0.5 \ \Omega \angle -90°)$
$= 2.121 \text{ V} \angle -150°$

and $\quad v = \sqrt{2}(2.121) \sin(\omega t - 150°) = \mathbf{3.0 \sin(\omega t - 150°)}$



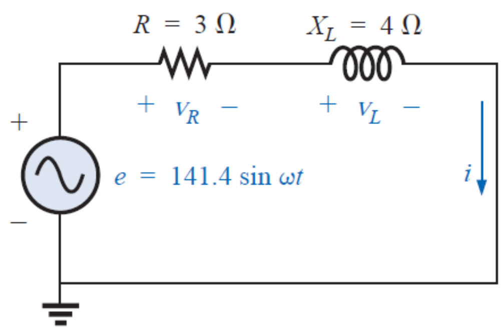


13.5 Series Configuration



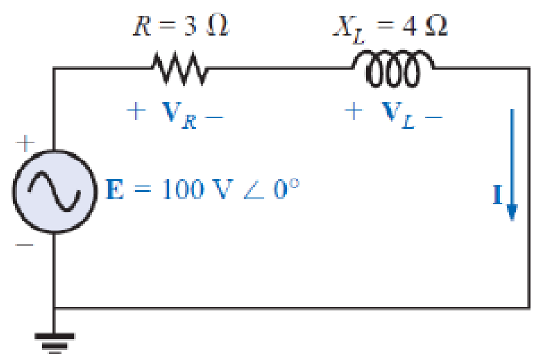
$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N$$

EXAMPLE: Determine the input impedance to the series network and find I , Vr,VL. Draw the impedance diagram.



Solutions:

$$e = 141.4 \sin \omega t \Rightarrow \mathbf{E} = 100 \text{ V} \angle 0^\circ$$



$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 3\ \Omega \angle 0^\circ + 4\ \Omega \angle 90^\circ = 3\ \Omega + j4\ \Omega$$

$$\mathbf{Z}_T = 5\ \Omega \angle 53.13^\circ$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{100 \text{ V} \angle 0^\circ}{5\ \Omega \angle 53.13^\circ} = 20 \text{ A} \angle -53.13^\circ$$

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (20 \text{ A} \angle -53.13^\circ)(3\ \Omega \angle 0^\circ)$$
$$= 60 \text{ V} \angle -53.13^\circ$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (20 \text{ A} \angle -53.13^\circ)(4\ \Omega \angle 90^\circ)$$
$$= 80 \text{ V} \angle 36.87^\circ$$