Examples 2

R.C.L series

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EXAMPLE: Sketch the following complex numbers in the complex plane:

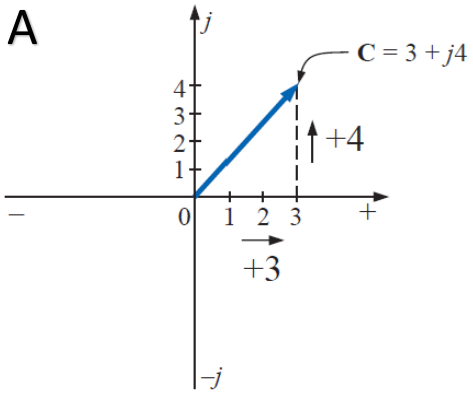
a) $\mathbf{C} = 3 + j\,4$

b) $\mathbf{C} = 0 - j\,6$

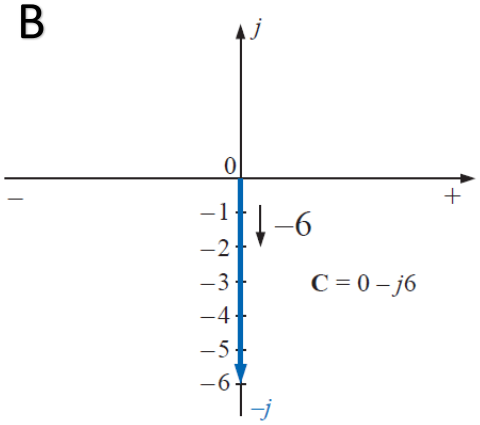
c) $\mathbf{C} = -10 - j20$

Solutions:

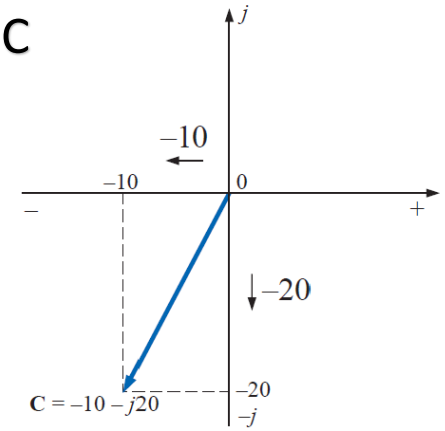
A



B



C



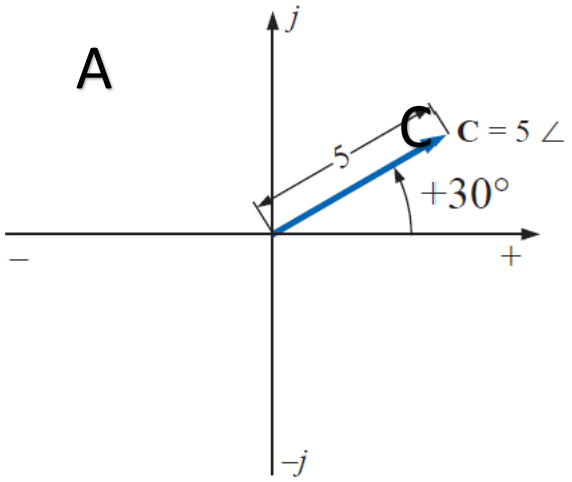
EXAMPLE: Sketch the following complex numbers in the complex plane:

a) $\mathbf{C} = 5 \angle 30°$

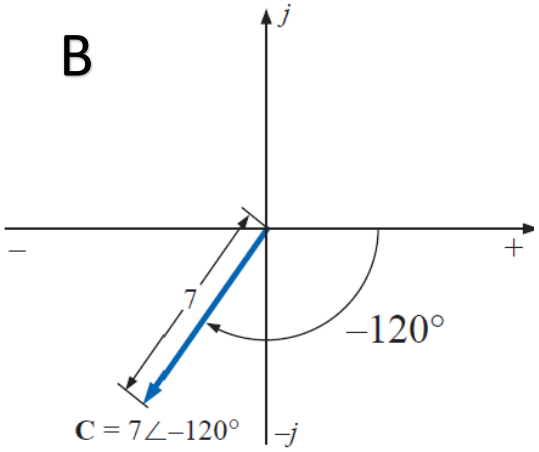
b) $\mathbf{C} = 7 \angle -120°$

c) $\mathbf{C} = -4.2 \angle 60°$

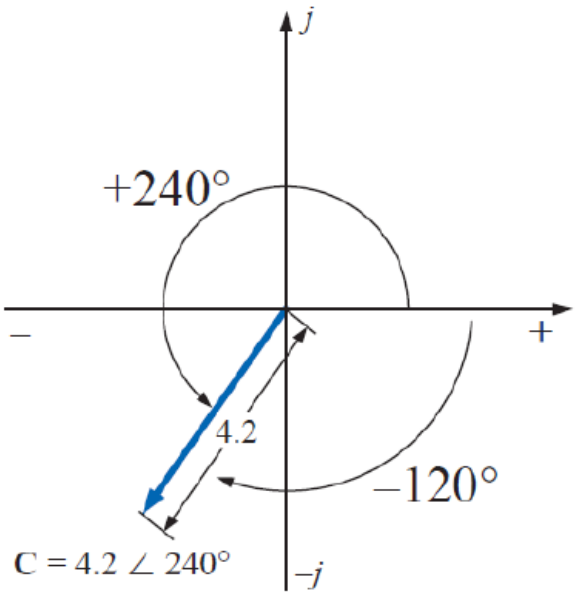
A



B



$$\mathbf{C} = -4.2 \angle 60° = 4.2 \angle 60° + 180°$$
$$= 4.2 \angle +240°$$



EXAMPLE: Convert the following from rectangular to polar form:

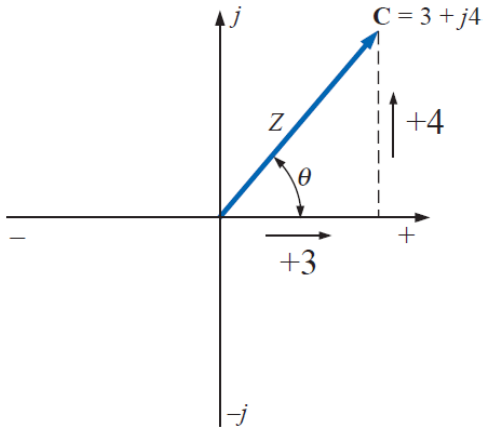
$$\mathbf{C} = 3 + j\,4$$

Solutions:

$$Z = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

$$\mathbf{C = 5 \angle 53.13^\circ}$$



EXAMPLE: Convert the following from polar to rectangular form:

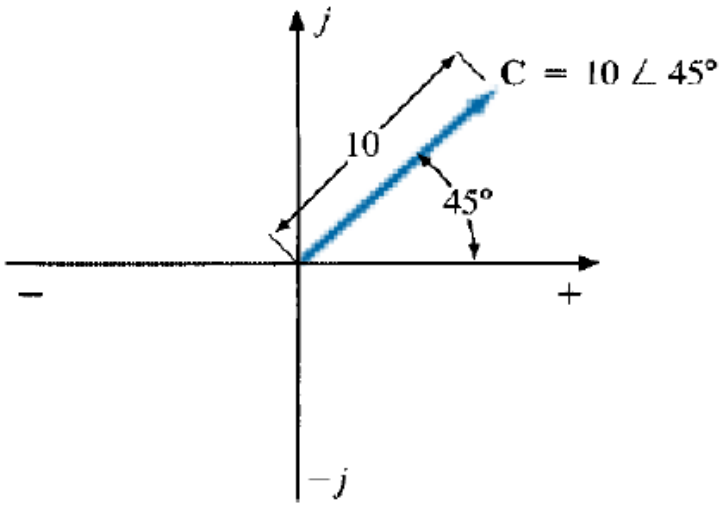
$$\mathbf{C} = 10 \angle 45^\circ$$

Solutions:

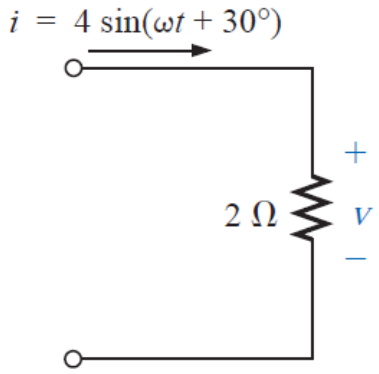
$$X = 10\cos 45^\circ = (10)(0.707) = 7.07$$

$$Y = 10\sin 45^\circ = (10)(0.707) = 7.07$$

$$\mathbf{C = 7.07 + j7.07}$$



EXAMPLE: find the voltage v for the circuit. Sketch the waveforms of v and i.

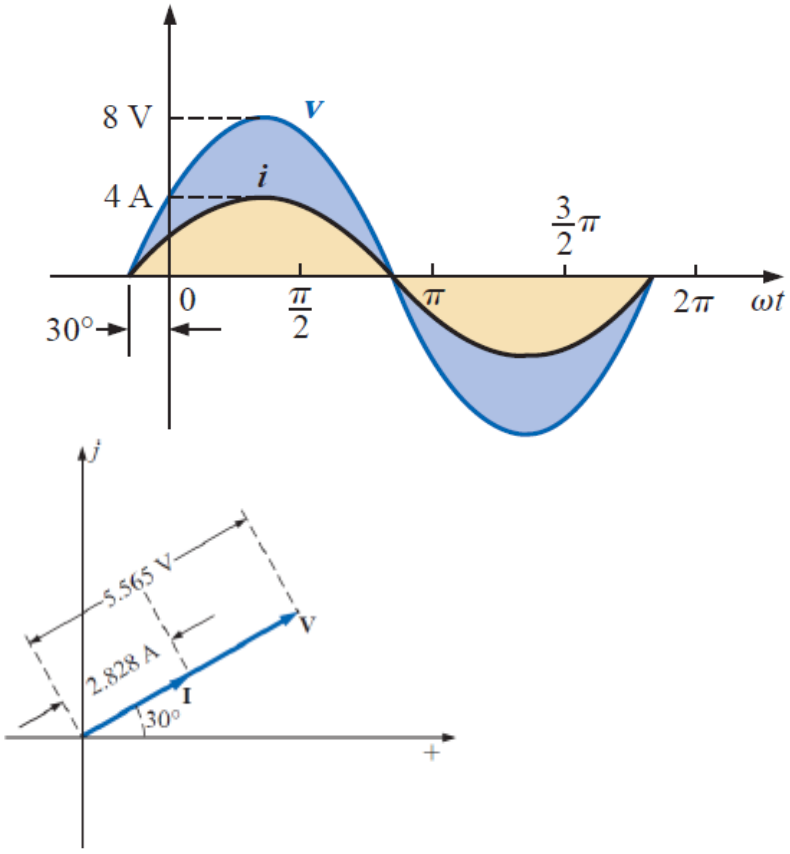


Solutions:

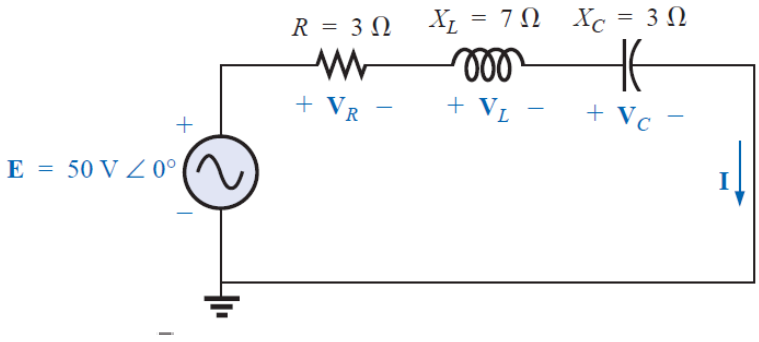
$$i = 4\sin(\omega t + 30°) \Rightarrow \text{phasor form } \mathbf{I} = 2.828\text{ A} \angle 30°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_R = (I \angle \theta)(R \angle 0°) = (2.828\text{ A} \angle 30°)(2\ \Omega \angle 0°)$$
$$= 5.656\text{ V} \angle 30°$$

$$v = \sqrt{2}(5.656)\sin(\omega t + 30°) = \mathbf{8.0\ \sin(\omega t + 30°)}$$



EXAMPLE: Determine the input impedance to the series network and find i , Vr, Vc ,VL. Draw the impedance diagram.



Solutions:

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 = R\angle 0^\circ + X_L\angle 90^\circ + X_C\angle -90^\circ$$
$$= 3\ \Omega + j\,7\ \Omega - j\,3\ \Omega = 3\ \Omega + j\,4\ \Omega$$

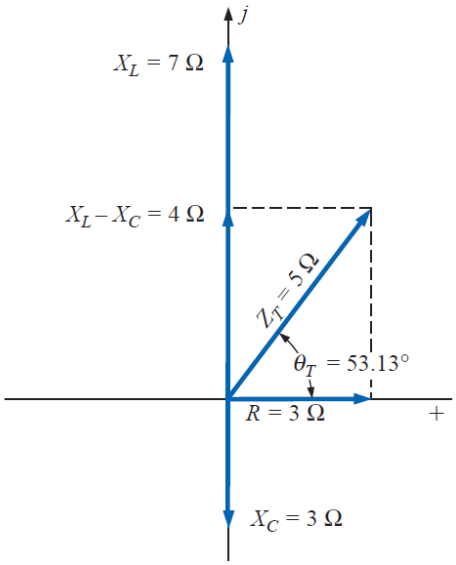
$$\mathbf{Z}_T = \mathbf{5\ \Omega\angle 53.13^\circ}$$

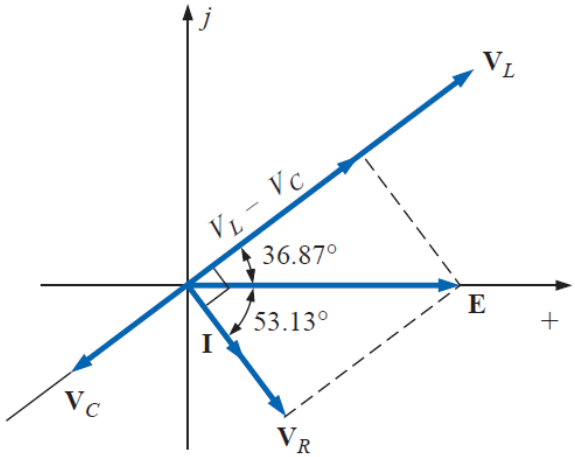
$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\ \text{V}\angle 0^\circ}{5\ \Omega\angle 53.13^\circ} = \mathbf{10\ \text{A}\angle -53.13^\circ}$$

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I\angle\theta)(R\angle 0^\circ) = (10\ \text{A}\angle -53.13^\circ)(3\ \Omega\angle 0^\circ)$$
$$= \mathbf{30\ \text{V}\angle -53.13^\circ}$$

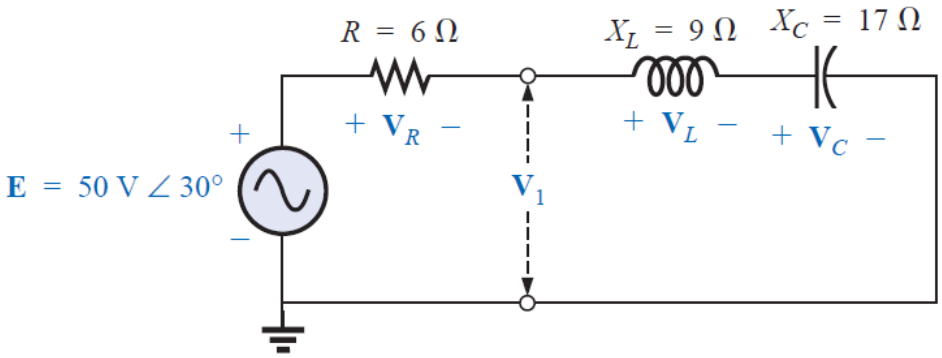
$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (I\angle\theta)(X_L\angle 90^\circ) = (10\ \text{A}\angle -53.13^\circ)(7\ \Omega\angle 90^\circ)$$
$$= \mathbf{70\ \text{V}\angle 36.87^\circ}$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I\angle\theta)(X_C\angle -90^\circ) = (10\ \text{A}\angle -53.13^\circ)(3\ \Omega\angle -90^\circ)$$
$$= \mathbf{30\ \text{V}\angle -143.13^\circ}$$





EXAMPLE: Using the voltage divider rule, find the unknown voltages VR, VL, VC, and V1 for the circuit.



Solutions:

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C} = \frac{(6\ \Omega\ \angle 0°)(50\ \text{V}\ \angle 30°)}{6\ \Omega\ \angle 0° + 9\ \Omega\ \angle 90° + 17\ \Omega\ \angle -90°}$$

$$= \frac{300\ \angle 30°}{6 + j\,9 - j\,17} = \frac{300\ \angle 30°}{6 - j\,8}$$

$$= \frac{300\ \angle 30°}{10\ \angle -53.13°} = \mathbf{30\ V\ \angle 83.13°}$$

$$\mathbf{V}_L = \frac{\mathbf{Z}_L\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90°)(50\ \text{V}\ \angle 30°)}{10\ \Omega\ \angle -53.13°} = \frac{450\ \text{V}\angle 120°}{10\ \angle -53.13°}$$

$$= \mathbf{45\ V\ \angle 173.13°}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(17\ \Omega\ \angle -90°)(50\ \text{V}\ \angle 30°)}{10\ \Omega\ \angle -53.13°} = \frac{850\ \text{V}\angle -60°}{10\ \angle -53°}$$

$$= \mathbf{85\ V\ \angle -6.87°}$$

$$\mathbf{V}_1 = \frac{(\mathbf{Z}_L + \mathbf{Z}_C)\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90° + 17\ \Omega\ \angle -90°)(50\ \text{V}\ \angle 30°)}{10\ \Omega\ \angle -53.13°}$$

$$= \frac{(8\ \angle -90°)(50\ \angle 30°)}{10\ \angle -53.13°}$$

$$= \frac{400\ \angle -60°}{10\ \angle -53.13°} = \mathbf{40\ V\ \angle -6.87°}$$